Kiểm tra khả năng chịu lực của cột bê tông cốt thép tiết diện hình chữ nhật sử dụng mặt tương tác ba chiều

TÓM TẮT

Khi thiết kế kết cấu cột bê tông cốt thép chịu nén lệch tâm xiên, người kỹ sư thường sử dụng các phép tính gần đúng quy về nén lệch tâm phẳng để tính toán lượng cốt thép cần thiết cho cột. Việc kiểm tra khả năng chịu lực của cột sau khi đặt cốt thép là cần thiết để xác định tính hợp lý của cốt thép được bố trí. Nghiên cứu này đã xây dựng các mặt tương tác ba chiều của cột bê tông cốt thép chịu nén lệch tâm xiên có mặt cắt ngang hình chữ nhật bằng phần mềm MATLAB, theo mô hình biến dạng phi tuyến. Mômen nội lực từ phần mềm như ETABS dùng để kiểm tra khả năng chịu lực của cột được xử lý đưa vào hệ số uốn dọc theo tiêu chuẩn TCVN 5574:2018, sau đó so sánh với mặt tương tác 3D đã thiết lập để đánh giá khả năng chịu lực của cột. Một ví dụ minh họa được trình bày trong bài viết này để làm tài liệu tham khảo cho sinh viên và kỹ sư kết cấu.

Keywords: mặt tương tác 3D, cột bê tông cốt thép, khả năng chịu lực, tiết diện chữ nhật, TCVN 5574:2018

Checking load bearing capacity of rectangular reinforced concrete columns using 3D interaction surfaces

ABSTRACT

In designing reinforced concrete column structures subjected to biaxial flexural force, engineers often use approximate calculations that refer to uniaxial flexural force to calculate the amount of reinforcement needed for the column. Checking the bearing capacity of the column after placing the reinforcement is necessary to determine the integrity of the placed reinforcement. This study built the 3D interaction surfaces of a reinforced concrete column subjected to biaxial flexural force with a rectangular cross-section using MATLAB software, according to the theory of non-linear material model. The internal moment from the software such as ETABS used for checking the column's capacity is processed to include the buckling factor according to TCVN 5574:2018, then compared with the established 3D interaction surface to evaluate the bearing capacity of the designed column. An example is illustrated in this article as a reference for students and structural engineers.

Keywords: *3D interaction surfaces, reinforced concrete columns, load-bearing capacity, rectangular cross-section, TCVN 5574:2018.*

1. INTRODUCTION

The required reinforcement for columns subjected to uniaxial flexural load is calculated based on Vietnamese national design standards TCVN 5574: 2018. However, this document does not mention the method for biaxial flexural force. Therefore, various approximated approaches were proposed. The superposition method, introduced by Moran, calculates reinforcement separately with (N, Mx) and (N, My) and then adds the results[1]. Symmetrically-reinforced rectangular sections can be designed to withstand an increased moment about one axis, as in BS 8110-1:2005 and Nguyen Dinh Cong[2]. Bresler[3] discussed other design criteria for short column, including three simple methods to generate the failure surfaces. From then, several authors have developed approximate methods based on the research of Bresler and ACI standards[4-6]. A combination of Bresler's equations and uniaxial interaction diagram based on TCVN 5574:2012 was proposed by Loan Nguyen[7] to estimate the required reinforcement or check the load-bearing capacity.

Existing software for generating interaction surfaces based on Vietnamese standards for biaxial flexural columns[8-10] has its limitations. Most software is written in Excel, with 2D graphs drawn according to the Mx-My or N-M relationship. For the new Vietnamese load and impact standards TCVN 2737:2023, the cases of dangerous internal forces that need to be considered have significantly increased. Checking the column's bearing capacity for each axial force or moment is time-consuming. Even imported software like ETABS and CSI-Col, which integrate the function of interaction surfaces, have drawbacks. They do not support Vietnamese standards, allow users to modify related coefficients, or provide visual load-bearing capacity checks. These limitations underscore the need for our program.

This research presents a novel program for generating 3D interaction surfaces of columns under biaxial flexural forces. Based on the limited strain and considering the buckling factor as per Vietnamese standard 5574: 2018, the method is presented in detail so that it can be regenerated. It also incorporates the ability to assess the load-bearing capacity of the columns. The step-by-step procedure for checking the load-bearing capacity corresponding to the internal forces calculated from the load combination is also detailed. In addition, the paper includes an illustrative example to demonstrate the program's capabilities further.

2. THE METHOD OF THE PROGRAM

2.1. Principles of the method

The strain distribution is linear across the section of the column (the cross-section of the column is always a plane).

The stress in the steel and concrete is given by the stress-strain graph as shown in Figure 1 and Figure 2.

The tensile resistance of the concrete is negligible.

The concrete strain at failure is 0.0035.

2.2. Stress-strain model of concrete

When $0 \le \varepsilon_b \le \varepsilon_{b1}$: $\sigma = E_b \varepsilon_b$ (1)

When $\varepsilon_{b1} < \varepsilon_b < \varepsilon_{b0}$:

$$\sigma_b = \left[\left(1 - \frac{\sigma_{b1}}{R_b} \right) \frac{\varepsilon_b - \varepsilon_{b1}}{\varepsilon_{b0} - \varepsilon_{b1}} + \frac{\sigma_{b1}}{R_b} \right] R_b \quad (2)$$

When $\varepsilon_{b0} \le \varepsilon_b \le \varepsilon_{b2}$: $\sigma_b = R_b$ (3)

Where: $\sigma_{b1} = 0.6 R_b$; $\varepsilon_{b1} = \sigma_{b1} / E_b$

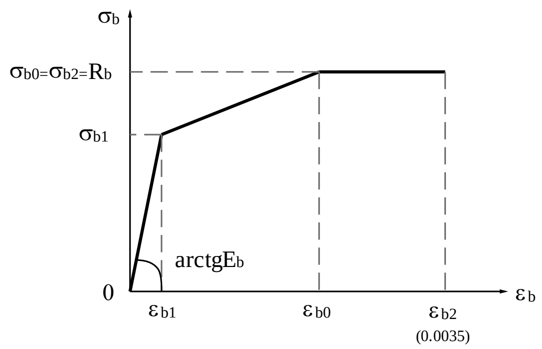


Figure 1. Stress-strain model of concrete

2.3. Stress-strain model of steel

When $0 \le \varepsilon_s < \varepsilon_{s0}$: $\sigma_s = \varepsilon_s E_s$ (4)

When $\varepsilon_{s0} \le \varepsilon_s \le \varepsilon_{s2}$: $\sigma_s = R_s$ (5)

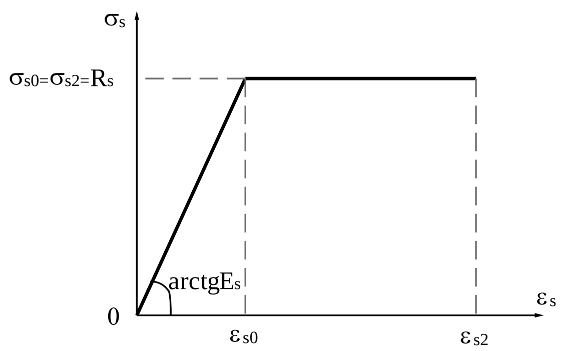


Figure 2. Stress-strain model of steel

2.4. Generation of biaxial interaction surfaces

Considering the section of a reinforced concrete column with pre-arranged reinforcement, the Oxy axis system with the center O coincides with the coordinates of the column section's center of gravity. Each steel bar is modeled by a circle with diameter φ and area A_s. The concrete part is discretized into a matrix of equal elements. Each element in the matrix is a square with size du and area du^2. The size of these elements is relatively small, so it can be assumed that the stress in the elements is considered evenly distributed within that element. The coordinates of steel bars and concrete elements are (x_{si}, y_{si}), and (x_{bi}, y_{bi}), respectively, as shown in Figure 3.

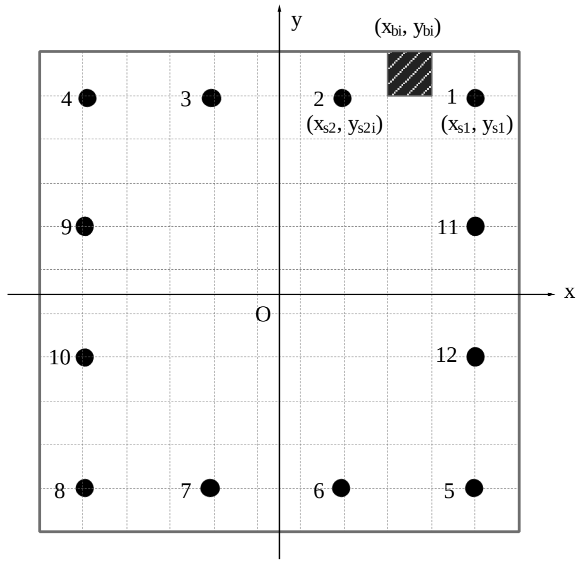


Figure 3. Concrete elements and steel bar coordinates

Assuming the neutral axis's occurrence of three cases, as shown in Figure 4. Figure 4a shows the neutral axis inclined (biaxial flexural compression), Figure 4b shows the neutral axis parallel to the x-axis (uniaxial flexural compression in the y-direction), and Figure 4c shows the neutral axis parallel to the y-axis (uniaxial flexural compression in the x-direction).

The neutral axis' position was changed to cover all the possible scenarios for the column. In this research, the interaction surfaces were constructed for the first quadrant, assuming that the maximum compressive strain of concrete is in the top-right corner, equal to 0.0035.

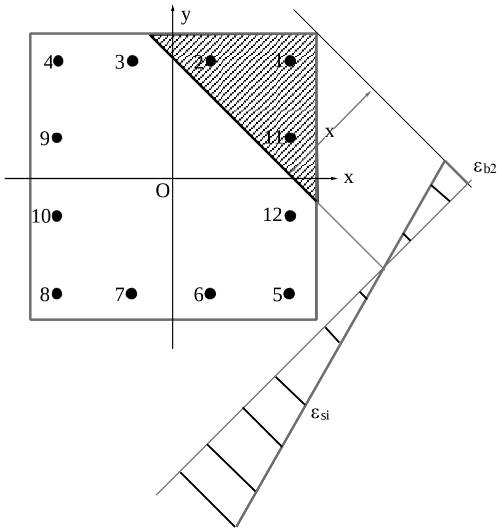
The strain of each steel and concrete element is calculated according to Equation (6).

$$\varepsilon_i = \frac{h_{0i} - x}{x} \varepsilon_{b2} \quad (6)$$

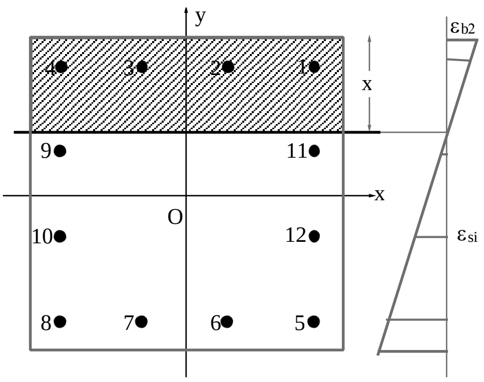
Where:

x is the distance between the neutral axis and the top-right corner.

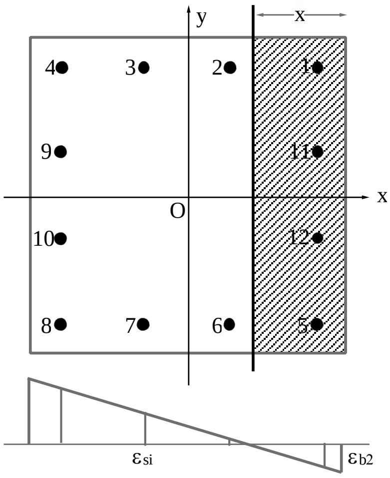
h_{0i} is the distance between the material element and the straight line paralleled to the neutral axis across the farthest point of the compression zone, as shown in Figure 5. h_{0i} can be calculated from the coordinates (xi, yi) of steel bars or concrete elements and the equation of neutral axis.



(a)



(b)



(c)

Figure 4. Neutral axis positions and strain distribution

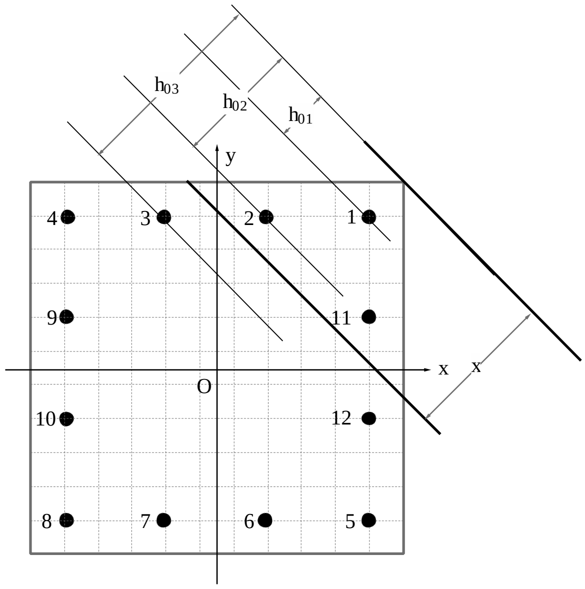


Figure 5. h_{0i} in the general scenario of an inclined neutral axis

Then, the stress in each steel bar and concrete element is calculated per the stress-strain model in 2.2 and 2.3.

Assume that:
Compression stresses in concrete are positive values;
Compression stresses in steels have negative values, and tensile stresses are positive;
N_z is positive when the column is compressed;
N_z, M_x, M_y on the failure surfaces was calculated as below:

$$N_z = -\sum_{i=1}^{n} \sigma_{si} A_{si} + \sum_{i=1}^{m} \sigma_{bi} d_u^2 \quad (7)$$

$$M_x = -\sum_{i=1}^{n} \sigma_{si} A_{si} y_{si} + \sum_{i=1}^{m} \sigma_{bi} d_u^2 y_{bi} \quad (8)$$

$$M_y = -\sum_{i=1}^{n} \sigma_{si} A_{si} x_{si} + \sum_{i=1}^{m} \sigma_{bi} d_u^2 x_{bi} \quad (9)$$

Where n is the number of steel bars, m is the number of concrete elements.
The maximum compression force columns can withstand is N_{max}, which occurs in concentric compression. When accounting for the buckling factor, N_{max} must be decreased by a coefficient φ, whose values change linearly from 0.9 to 0.85 when L_0/h varies from 10 to 20.

$$N_{\max} = \varphi\left(R_s \sum_{i=1}^{n} A_{si} + R_b bh \right) \quad (10)$$

Where:

b, h are the dimensions of rectangle cross-section;

L_0 is the effective length of the column.

The interaction surface includes a series of discrete points, as shown in the Figure 6.

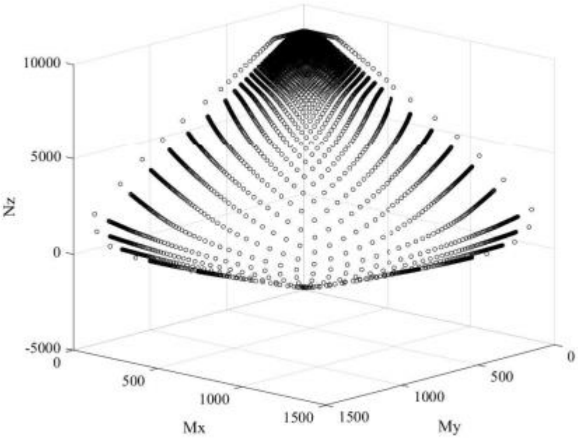


Figure 6. Discrete points of the interaction surface

The space defined by the interaction surface is called column capacity interaction volume.

3. CHECK COLUMN CAPACITY

The designer must check the capacity of columns for every loading combination at every output station. To check a specific column for a particular loading combination at a specific location, the program follows a set of steps.

(1) Input the internal moments and forces from the specified load cases and load combination, including N, M_{22} (or M_x), M_{33} (or M_y).

(2) Calculate the eccentricity of axial force.

$$e_{1x} = \frac{M_x}{N};\ e_{1y} = \frac{M_y}{N} \tag{11}$$

(3) Define the random eccentricity e_a.

$$\begin{aligned} e_{ax} &= min\ \left(L/600,\ b/30,\ 10\ mm\right) \\ e_{ay} &= min\ \left(L/600,\ h/30,\ 10\ mm\right) \end{aligned} \tag{12}$$

Where L is the length of the column.

(4) Define the initial eccentricity e_0.

$$e_{0x} = min\ \left(e_{1x}, e_{ax}\right);\ e_{0y} = min\ \left(e_{1y}, e_{ay}\right) \tag{13}$$

(2) Determine the coefficient η_x and η_y due to slender column effect.

$$\eta_x = \frac{1}{1-\dfrac{N}{N_{crx}}};\ \eta_y = \frac{1}{1-\dfrac{N}{N_{cry}}} \tag{14}$$

$$N_{crx} = \frac{\pi^2 D_x}{L_0^2};\ N_{cry} = \frac{\pi^2 D_x}{L_0^2} \tag{15}$$

$$\begin{aligned} D_x &= k_{bx} E_b I_x + k_s E_s I_{sx} \\ D_y &= k_{by} E_b I_y + k_s E_s I_{sy} \end{aligned} \tag{16}$$

Where:

I_x, I_y are the inertia moments of the cross-section,

I_{sx}, I_{sy} are the inertia moments of the steel bars,

$k_s = 0.7$

$$k_{bx} = \frac{0.15}{\varphi_{Lx}\left(0.3+\delta_e\right)};\ k_{by} = \frac{0.15}{\varphi_{Ly}\left(0.3+\delta_e\right)} \tag{17}$$

$$\varphi_{Lx} = 1 + \frac{M_{L1x}}{M_{Lx}};\ \varphi_{Ly} = 1 + \frac{M_{L1y}}{M_{Ly}} \tag{18}$$

$$\delta_{ex} = e_{0x}/b\ ;\ \delta_{ey} = e_{0y}/h\ ;\ 0.15 \le \delta_e \le 1.5 \tag{19}$$

M_L is the internal moment of total load and M_{L1} is the internal moment of dead load about the axis across the farthest tension steel bar. In the most dangerous cases, φ_L is equal to 2.

(3) Determine total design moments by multiplying the coefficient e_x and e_y by the factored axial forces N_z obtained from the analysis.

$$e_x = e_{0x}\eta_x, e_y = e_{0y}\eta_y \tag{20}$$

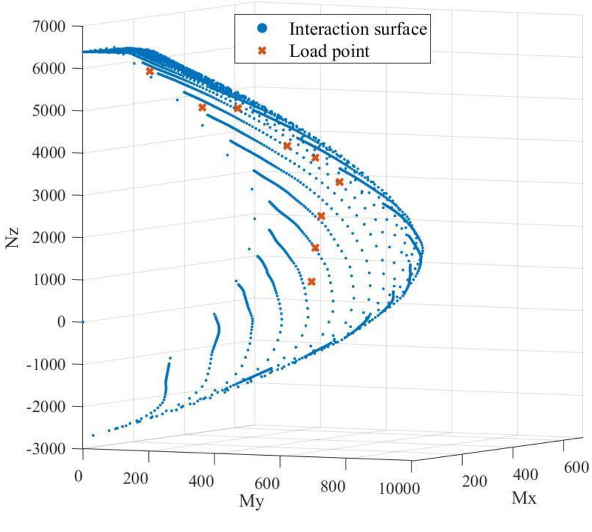
$$M_x^* = Ne_x; M_y^* = Ne_y \tag{21}$$

(4) Determine whether the point, defined by the resulting axial load and biaxial moment set (N, M_x^*, M_y^*) lies within the interaction volume.

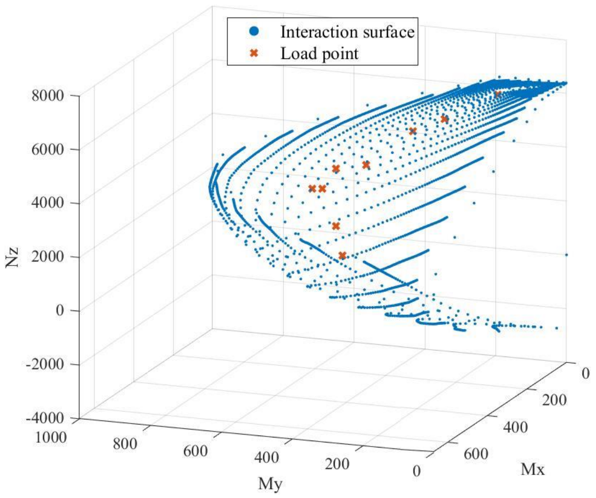
There are different ways to examine whether the load points lie within the interaction volume. Beside using a 3D rotational interaction surface, this program uses the method mentioned below.

First, calculate the neutral axis angle of each load point. Second, the N - M interaction diagram is determined by cutting the interaction surface at the given angles. Then, the axial load N is used to determine the resultant moment on the N-M curve. Finally, if Equation 22 is true, the column can withstand the load (Figure 8-9).

$$M^* = \sqrt{\left(M_x^*\right)^2 + \left(M_y^*\right)^2} \le \text{Resultant } M \tag{22}$$



a)



b)

Figure 7. 3D Rotational Interaction Surface

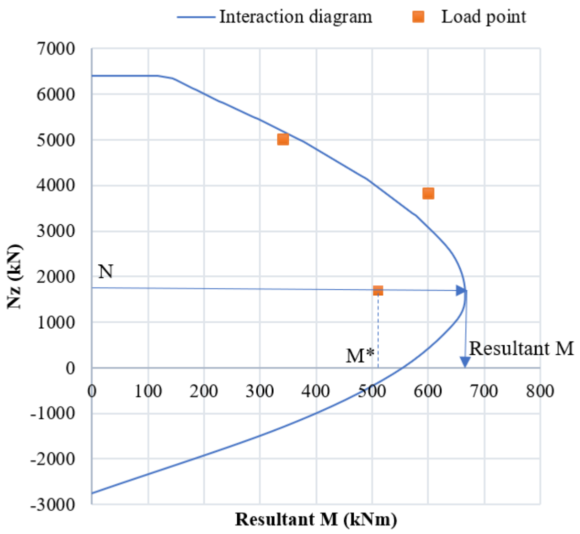


Figure 8. Load point inside the interaction volume

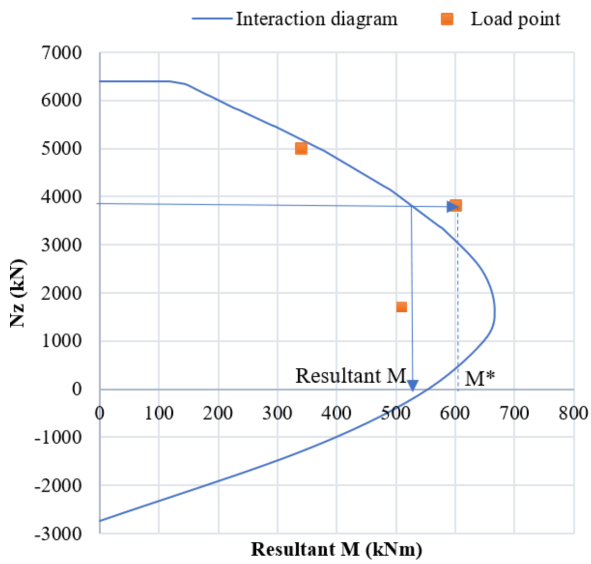


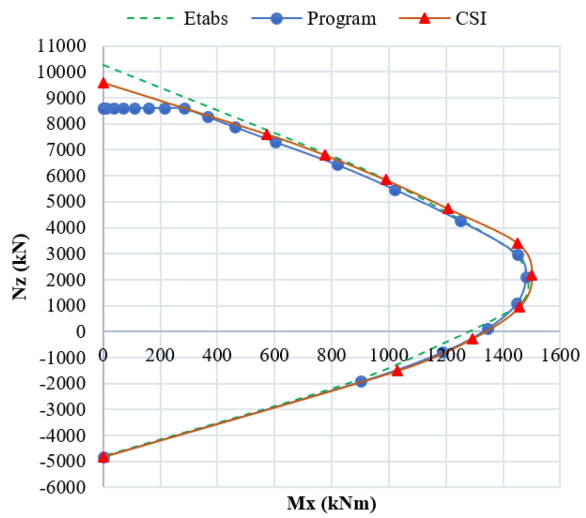
Figure 9. Load point outside the interaction volume

4. MODEL VERIFYING

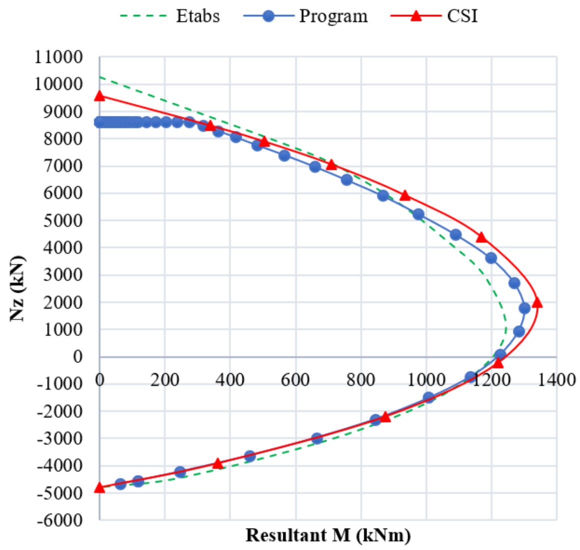
The 3D model generated by the proposed program is verified in comparison to the result from ETABS ver18.1.1 and CSI-Col ver10.0.0.

4.1. Example 1

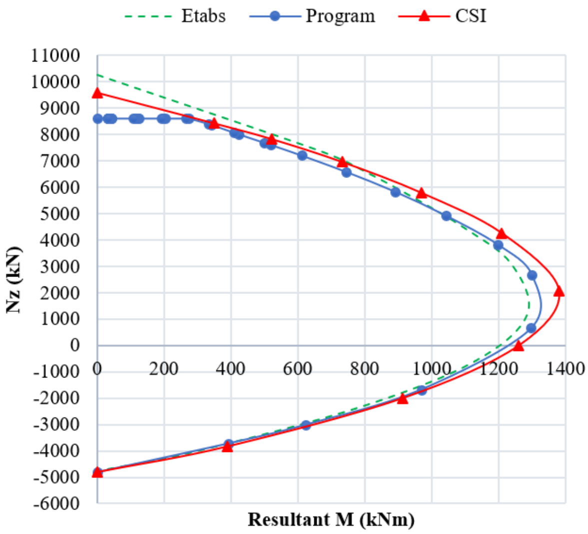
A column has a square cross-section of 70 cm x 70 cm, concrete grade B20, reinforcement CB400-V, 28 Ø 25 mm arranged evenly along the perimeter. The distance between the edge and the central of steel bars a_0 = 4 cm. The length of the column is 7 m. Work condition coefficient of concrete γ_b = 0.85.



a) 0^0



b) 45⁰



c) 60⁰

Figure 10. Interaction diagram of square column

The proposed program, ETABS, and CSI-Col give the same results, with an error of less than 4%. There are no noticeable differences in the uniaxial scenario. The maximum error occurs at the balance point, the transition between compression-controlled and tension-controlled points, in case the neutral axis angle is 60⁰. The discrepancy can happen because of the method for calculating the force and moment due to the concrete compression area, stress-strain relationships, failure criteria of concrete, and work condition coefficients.

4.2. Example 2

A column has rectangle cross section 0.7 m x 0.5 m, concrete grade B25, reinforcement CB400-V, 16 Ø 25 mm as shown in Figure 8. The distance between the edge and the central of steel bars a_0 = 4 cm. The length of the column is 4.5 m. Work condition coefficient of concrete γ_b = 0.85.

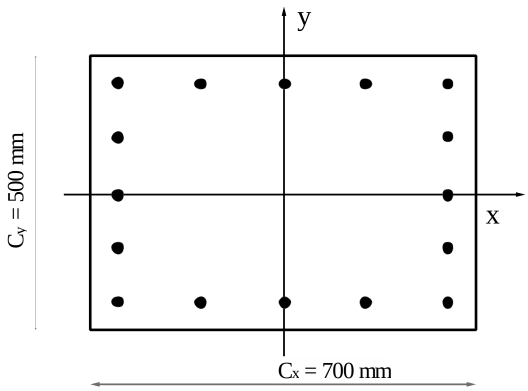
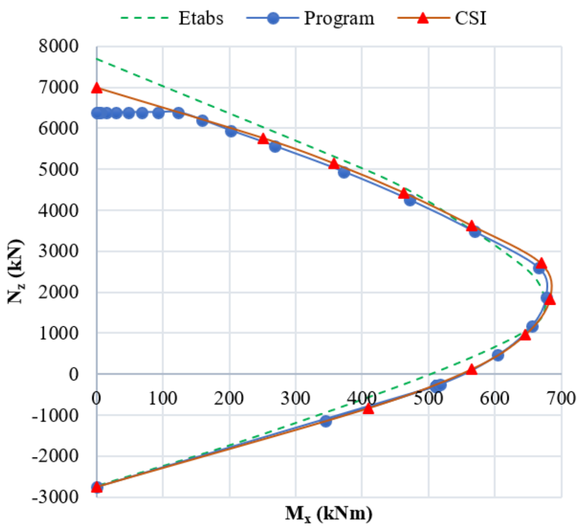
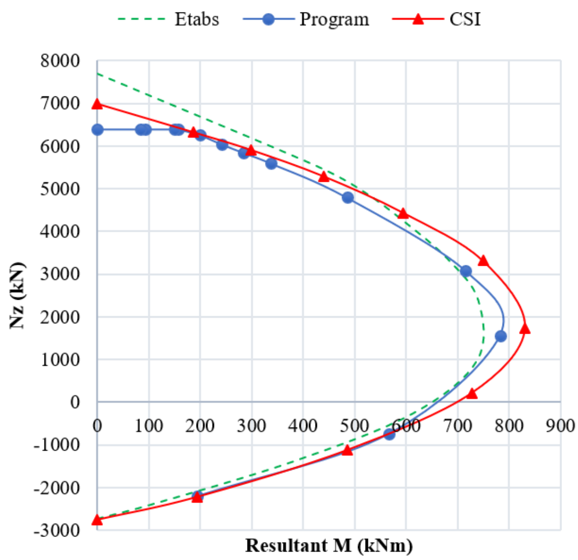


Figure 8. Rectangle column's cross-section



a) 0⁰



b) 45⁰

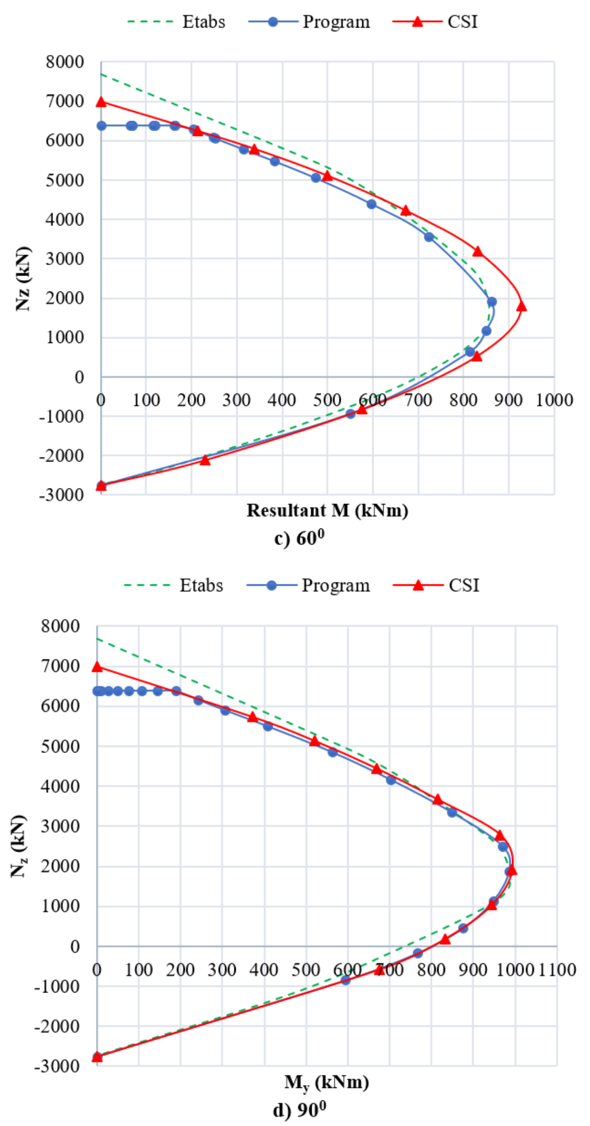


Figure 11. Interaction diagram of rectangle column

Notice: The relationship between the neutral axis angle and the point's coordinates is $\tan\alpha = \dfrac{My}{Mx}\dfrac{Ix}{Iy}$.

The features of the results are the same as in Example 1. When the neutral axis angle is 0^0 and 90^0, the discrepancies between the three software are negligible. The max error occurs in the same case and is less than 7.2%.

Through the above examples, compared with other research[9,10], the proposed program is reliable and it can be use in practical design, research and teaching.

5. ILLUSTRATIVE EXAMPLE

The column is as shown in Example 2 of 4.2. The internal moments and forces from the load combinations are given in Table 1. Internal moments of dead load are $M_{L1x} = 80$ kNm, $M_{L1y} = 300$ kNm.

Question: Checking the load-bearing capacity of the column.

Table 1. Forces and moments from load combinations.

Combo	N (kN)	M_x(kNm)	M_y(kNm)
1	5400	90	315
2	5310	180	360
3	6300	45	180
4	4320	270	450
5	4050	270	540
6	2700	180	630
7	1800	270	540
8	900	315	495
9	3420	315	585

The load points are determined from Equations 11-21, as shown in Table 2.

Table 2. Load points after considering slender column effect

Combo	N (kN)	M^*_x (kNm)	M^*_y (kNm)	M* (kNm)
1	5400	134.7	326.1	352.8
2	5310	190.3	372.2	418.1
3	6300	162.2	188.4	248.6
4	4320	282.3	462.3	541.7
5	4050	281.6	553.8	621.3
6	2700	185.5	641.6	667.9
7	1800	275.7	547.2	612.8
8	900	319.0	498.9	592.2
9	3420	326.6	598.0	681.4

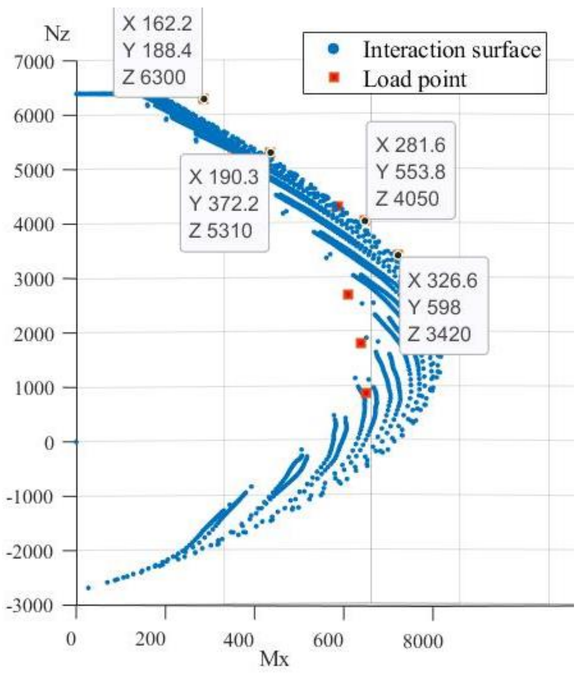


Figure 12. Checking load-bearing capacity using 3D

interaction surface

Using 3D interaction surface, the program points out 4 load points outside of the interaction volume, as shown in Figure 12. They come from combination 2, 3, 5 and 9.

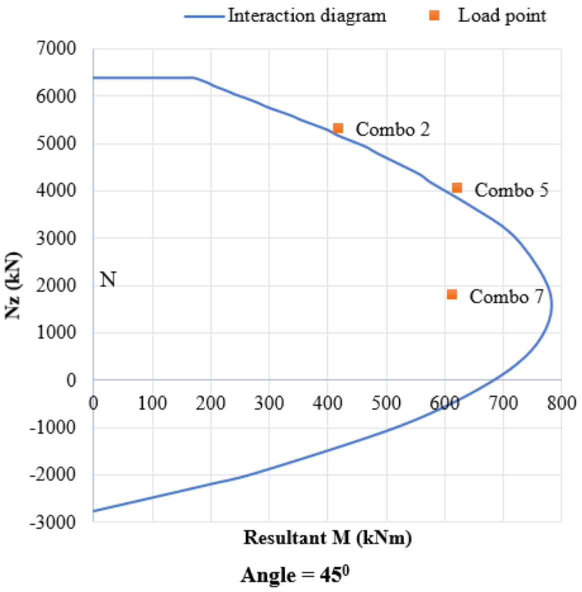


Figure 13. Checking load-bearing capacity using N-M interaction diagram (45^0)

The integrated module in the program to checking the load-bearing capacity using N-M curve yields the result in Table 3.

Table 3. Checking load-bearing capacity using N-M interaction diagram

Com	M* (kNm)	Angle (degree)	Resultant M	Check
1	352.8	51	386.1	Pass
2	418.1	45	395.8	Fail
3	248.6	31	174.3	Fail
4	541.7	40	542.4	Pass
5	621.3	45	594.0	Fail
6	667.9	60	815.7	Pass
7	612.8	45	776.6	Pass
8	592.2	39	720.9	Pass
9	681.4	43	664.9	Fail

As can be seen from Figure 12 and Table 3, the two methods of checking load-bearing capacity show identical results.

5. CONCLUSION

This study introduces a program that generates 3D interaction surfaces of columns subjected to biaxial flexural forces. The method is explained in detail, using limited strain and considering the buckling factor per Vietnamese standard 5574: 2018. The program also includes a feature to evaluate the load-bearing capacity of the columns.

The proposed method and program are highly accurate compared to imported software such as ETABS and CSI-Col. In addition, the program can accurately and fast check column capacity subjected to various biaxial load cases based on TCVN 5574:2018.

The proposed program can be used for further research, including investigating the effects of initial conditions on the column capacity, such as uncertainty of materials properties, effective cover, work condition coefficients, stress-strain relationship, etc. However, there is a requirement for a number of experiments to verify the precision of the program.

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