Xây dựng mặt tương tác và đánh giá khả năng chịu lực của cột bê tông cốt thép tiết diện hình chữ nhật

TÓM TẮT

Khi thiết kế kết cấu cột bê tông cốt thép chịu nén lệch tâm xiên, việc kiểm tra khả năng chịu lực của cột sau khi đặt cốt thép là cần thiết để xác định tính hợp lý của cốt thép được bố trí. Nghiên cứu này đã xây dựng các mặt tương tác ba chiều của cột bê tông cốt thép chịu nén lệch tâm xiên có mặt cắt ngang hình chữ nhật bằng phần mềm MATLAB, theo mô hình biến dạng phi tuyến. Mômen nội lực từ phần mềm như ETABS dùng để kiểm tra khả năng chịu lực của cột được xử lý đưa vào hệ số uốn dọc theo tiêu chuẩn TCVN 5574:2018, sau đó so sánh với mặt tương tác 3D đã thiết lập để đánh giá khả năng chịu lực của cột. Một ví dụ minh họa được trình bày trong bài viết này để làm tài liệu tham khảo cho sinh viên và kỹ sư kết cấu.

Keywords: *mặt tương tác 3D, cột bê tông cốt thép, khả năng chịu lực, tiết diện chữ nhật, TCVN 5574:2018*

Generating 3D interaction surfaces and assessing load-bearing capacity of rectangular reinforced concrete columns

ABSTRACT

In designing reinforced concrete columns subjected to combined axial load and biaxial bending, checking the bearing capacity of the column after placing the reinforcement is necessary to determine the integrity of the placed reinforcement. This study developed a tool to generate the 3D interaction surfaces of rectangular reinforced concrete columns subjected to biaxial flexural and axial loads using MATLAB software, according to the theory of non-linear material model. The internal moments from the software such as ETABS is processed to include the buckling factor according to TCVN 5574:2018, then compared with the established 3D interaction surface to assessing the bearing capacity of the designed column. An example is illustrated in this article as a reference for students and structural engineers.

Keywords: *3D interaction surfaces, reinforced concrete columns, load-bearing capacity, rectangular cross-section, TCVN 5574:2018*

1. INTRODUCTION

The required reinforcement for columns subjected to uniaxial flexural and axial loads is calculated based on Vietnamese national design standards TCVN 5574: 2018. However, the method for biaxial flexural and compression forces is sophisticated for practical application. Various approximated approaches were proposed. The superposition method, introduced by Moran, calculates reinforcement separately with (*N*, *Mx*) and (*N*, *My*) and then adds the results[1]. Symmetrically-reinforced rectangular sections can be designed to withstand an increased moment about one axis, as detailed in BS 8110-1:2005 and Nguyen[2]. Bresler[3] discussed other design criteria for short columns, including three simple methods to generate the failure surfaces. From then, several authors have developed approximate methods based on the research of Bresler and ACI standards[4-6]. In particular, Mavichak and Furlong[5] conducted experiments on nine rectangular cross-section columns and compared the test data with an analytical model to validate the equation for strength approximations. Some authors suggested methods to draw 2D interaction diagrams based on TCVN 5574:2012[7-9]. A combination of Bresler's equations and uniaxial interaction diagrams based on TCVN 5574:2012 was proposed by Nguyen[10] to estimate the required reinforcement or check the load-bearing capacity. Nguyen[11] presented the interaction diagram of circular reinforced concrete columns under fire, based on TCVN5574:2012 and Eurocode 2. Hoach[12] and Tran et al.[13] developed software for creating interaction diagrams based on TCVN 5574:2018 for biaxially loaded members. Existing software for generating interaction surfaces based on Vietnamese standards for biaxial flexural columns has its limitations. Most software is written in Excel, with 2D graphs drawn according to the *Mx*-*My* or *N*-*M* relationship. For the new Vietnamese load and impact standards TCVN 2737:2023, the cases of dangerous internal forces that need to be considered have significantly increased. Checking the column's bearing capacity for each axial force or moment is time-consuming. Pham[14] proposed approximate approaches, which refer to uniaxial compression, to create the interaction diagram and check the load-bearing capacity for columns under eccentric compression based on TCVN 5574:2012. Hoach[15] and Nguyen[16] explained a method to calculate the coefficient of safety for reinforced concrete columns, considering the slenderness of columns according to TCVN 5574:2018. The proposed method to identify safety factors requires discrete points of the interaction surface to be displayed in a spherical coordinate system.

Even imported software like ETABS and CSI-Col, which integrate the function of drawing interaction surfaces, have drawbacks. They do not support Vietnamese standards, allow users to modify related coefficients, or provide visual load-bearing capacity checks. These limitations underscore the need for our program.

This research presents a novel program, written by MATLAB, for generating 3D interaction surfaces of columns under biaxial flexural and axial forces. The program is based on the limited strain and non-linear material models, considering the buckling factor per Vietnamese standard 5574:2018. It also includes a tool for assessing the load-bearing capacity of the columns. The paper provides a step-by-step procedure for checking the load-bearing capacity corresponding to the internal forces calculated from the load combination. Additionally, the paper includes an illustrative example to demonstrate the program's capabilities further.

2. METHODOLOGY

2.1. Assumptions

The strain distribution is linear across the section of the column (the cross-section of the column is always a plane).

The stress in the steel and concrete is given by the stress-strain graphs as shown in Figure 1 and Figure 2.

The tensile resistance of the concrete is negligible.

The concrete strain at failure is 0.0035.

2.2. Stress-strain model of concrete

According to TCVN 5574:2018, for normal-strength concrete, the stress-strain graph, which includes three or two segments, can be used when calculating reinforced concrete per a non-linear deformation model. In this research, a three-segment stress-strain graph is selected.

When $0 \le \varepsilon_b \le \varepsilon_{b1}$: $\sigma = E_b \varepsilon_b$ (1)

When $\varepsilon_{b1} < \varepsilon_b < \varepsilon_{b0}$:

$$\sigma_b = \left[\left(1 - \frac{\sigma_{b1}}{R_b}\right)\frac{\varepsilon_b - \varepsilon_{b1}}{\varepsilon_{b0} - \varepsilon_{b1}} + \frac{\sigma_{b1}}{R_b}\right] R_b \quad (2)$$

When $\varepsilon_{b0} \le \varepsilon_b \le \varepsilon_{b2}$: $\sigma_b = R_b$ (3)

Where: $\sigma_{b1} = 0.6R_b$; $\varepsilon_{b1} = \sigma_{b1} / E_b$

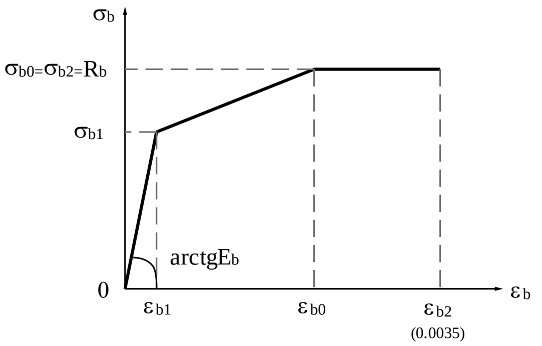


Figure 1. Stress-strain model of concrete

2.3. Stress-strain model of steel

According to TCVN 5574:2018, it is possible to use a simplified stress-strain chart with two segments for the steel, which has a yield point, as described below.

When $0 \le \varepsilon_s < \varepsilon_{s0}$: $\sigma_s = \varepsilon_s E_s$ (4)

When $\varepsilon_{s0} \le \varepsilon_s \le \varepsilon_{s2}$: $\sigma_s = R_s$ (5)

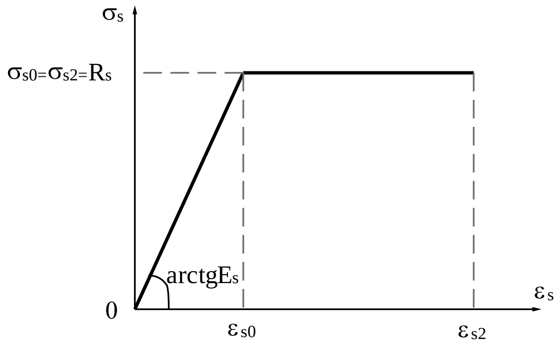


Figure 2. Stress-strain model of steel

The tensile and compressive stress-strain graphs of steel are identical.

2.4. Generation of 3D interaction surfaces

Considering the section of a reinforced concrete column with pre-arranged reinforcement, the Oxy axis system with the center O coincides with the column section's center of gravity. Each steel bar is modeled by a circle with diameter Ø and area A_s. The concrete part is discretized into a matrix of equal elements. Each element in the matrix is a square with size *du* and area du^2. The size of these elements is relatively small, so it can be assumed that the stress in the elements is considered evenly distributed within that element. The coordinates of steel bars and concrete elements are (x_{si}, y_{si}), and (x_{bi}, y_{bi}), respectively, as shown in Figure 3.

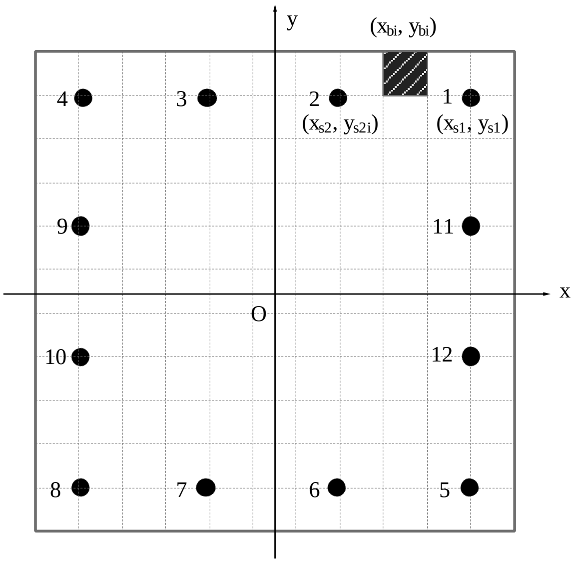


Figure 3. Concrete elements and steel bar coordinates

Assume the neutral axis occurs in three cases, as shown in Figure 4. Figure 4a shows the neutral axis inclined (biaxial flexural compression), Figure 4b shows the neutral axis parallel to the x-axis (uniaxial flexural compression in the y-direction), and Figure 4c shows the neutral axis parallel to the y-axis (uniaxial flexural compression in the x-direction).

The neutral axis' angle and distance were changed to cover all the possible scenarios for the column. In this research, the interaction surfaces were constructed for the first quadrant, assuming that the maximum compressive strain of concrete is in the top-right corner, equal to 0.0035.

The strain of each steel and concrete element is calculated according to Equation (6).

$$\varepsilon_i = \frac{h_{0i} - x}{x}\varepsilon_{b2} \tag{6}$$

Where:

x is the distance between the neutral axis and the top-right corner.

h_{0i} is the distance between the material element and the straight line paralleled to the neutral axis across the farthest point of the compression zone, as shown in Figure 5. h_{0i} can be calculated from the coordinates (xi, yi) of steel bars or concrete elements and the equation of neutral axis.

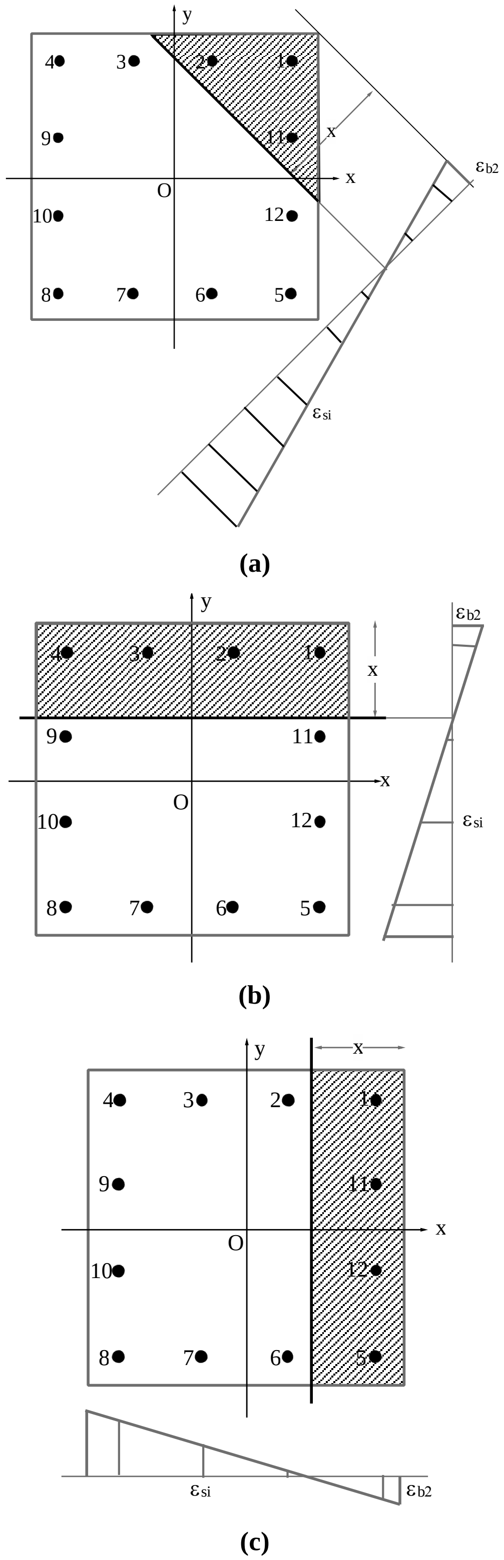


Figure 4. Neutral axis positions and strain distribution

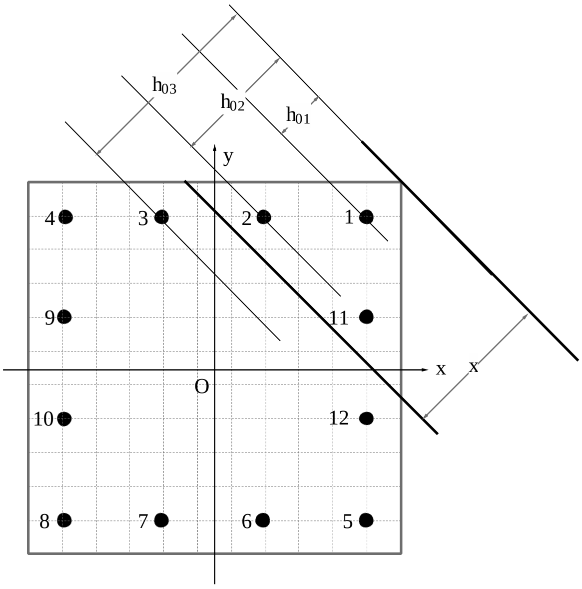


Figure 5. h_{0i} in the general scenario of an inclined neutral axis

Then, the stress in each steel bar and concrete element is calculated per the stress-strain model in 2.2 and 2.3.

Assume that:

Compression stresses in concrete are positive values;

Compression stresses in steels have negative values, and tensile stresses are positive;

N_{uz} is positive when the column is compressed;

N_{uz}, M_{ux}, M_{uy} on the failure surfaces was calculated as below:

$$N_{uz} = -\sum_{i=1}^{n} \sigma_{si} A_{si} + \sum_{i=1}^{m} \sigma_{bi} d_u^2 \tag{7}$$

$$M_{ux} = -\sum_{i=1}^{n} \sigma_{si} A_{si} y_{si} + \sum_{i=1}^{m} \sigma_{bi} d_u^2 y_{bi} \tag{8}$$

$$M_{uy} = -\sum_{i=1}^{n} \sigma_{si} A_{si} x_{si} + \sum_{i=1}^{m} \sigma_{bi} d_u^2 x_{bi} \tag{9}$$

Where n is the number of steel bars, m is the number of concrete elements.

The maximum compression force columns can withstand is $N_{u,max}$, which occurs in concentric compression. At stated in TCVN 5574:2018, when accounting for the buckling factor, $N_{u,max}$ must be decreased by the coefficient φ, whose values change linearly from 0.9 to 0.85 when L_0/h varies from 10 to 20.

$$N_{u,\max} = \varphi \left(R_s \sum_{i=1}^{n} A_{si} + R_b bh \right) \tag{10}$$

Where:

b, h are the dimensions of the rectangular cross-section;

L_0 is the effective length of the column.

The interaction surface includes a series of discrete points, as shown in the Figure 6.

The space defined by the interaction surface is called column capacity interaction volume.

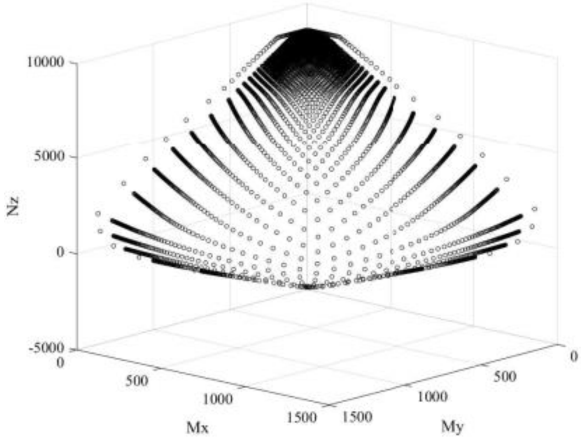


Figure 6. Discrete points of the interaction surface

2.5. Assessing column capacity

The designer must check the capacity of columns for every load combination at every output station. Based on TCVN 5574:2018, the program follows the flowing steps to check a specific column for a particular load combination at a specific location.

(1) Input the internal moments and forces from the specified load cases and load combination, including N, M_{22} (or M_x), M_{33} (or M_y).

(2) Calculate the eccentricity of axial force.

$$e_{1x} = \frac{M_x}{N};\ e_{1y} = \frac{M_y}{N} \tag{11}$$

(3) Define the random eccentricity e_a.

$$\begin{aligned} e_{ax} &= \min\left(L/600,\ b/30,\ 10\ mm\right) \\ e_{ay} &= \min\left(L/600,\ h/30,\ 10\ mm\right) \end{aligned} \tag{12}$$

Where L is the length of the column.

(4) Define the initial eccentricity e_0.

$$e_{0x} = \min\left(e_{1x}, e_{ax}\right);\ e_{0y} = \min\left(e_{1y}, e_{ay}\right) \tag{13}$$

(5) Determine the coefficient η_x and η_y due to slender column effect.

$$\eta_x = \frac{1}{1-\dfrac{N}{N_{crx}}};\ \eta_y = \frac{1}{1-\dfrac{N}{N_{cry}}} \tag{14}$$

$$N_{crx} = \frac{\pi^2 D_x}{L_0^2};\ N_{cry} = \frac{\pi^2 D_x}{L_0^2} \tag{15}$$

$$D_x = k_{bx} E_b I_x + k_s E_s I_{sx}$$
$$D_y = k_{by} E_b I_y + k_s E_s I_{sy} \tag{16}$$

Where:

I_x, I_y are the inertia moments of the cross-section,

I_{sx}, I_{sy} are the inertia moments of the steel bars,

$k_s = 0.7$

$$k_{bx} = \frac{0.15}{\varphi_{Lx}\left(0.3+\delta_e\right)};\quad k_{by} = \frac{0.15}{\varphi_{Ly}\left(0.3+\delta_e\right)} \tag{17}$$

$$\varphi_{Lx} = 1 + \frac{M_{L1x}}{M_{Lx}};\quad \varphi_{Ly} = 1 + \frac{M_{L1y}}{M_{Ly}} \tag{18}$$

$$\delta_{ex} = e_{0x}/b\ ;\ \delta_{ey} = e_{0y}/h\ ;\ 0.15 \le \delta_e \le 1.5 \tag{19}$$

M_L is the internal moment of total load and M_{L1} is the internal moment of dead load about the axis across the farthest tension steel bar. In the most dangerous cases, φ_L is equal to 2.

(6) Determine total design moments by multiplying the coefficient e_x and e_y by the factored axial forces N_z obtained from the analysis.

$$e_x = e_{0x}\eta_x, e_y = e_{0y}\eta_y \tag{20}$$

$$M_x^* = Ne_x; M_y^* = Ne_y \tag{21}$$

(7) Determine whether the point, defined by the resulting axial load and biaxial moment set (N, M_x^*, M_y^*) lies within the interaction volume.

There are different ways to examine whether the load points lie within the interaction volume. Beside using a 3D rotational interaction surface, this program uses the method mentioned below.

First, calculate the neutral axis angle of each load point.

$$\tan\alpha = \frac{M_x^*}{M_y^*}\frac{I_y}{I_x} \tag{22}$$

Second, cut the interaction surface at the above angles to obtain the *N-M* interaction diagram. Then, determine the resultant moment on the *N-M* curve corresponding to the axial force *N*. Finally, if Equation (23) is true, the column can withstand the load and vice versa (Figure 8-9).

$$M^* = \sqrt{\left(M_x^*\right)^2 + \left(M_y^*\right)^2} \le \text{Resultant } M \tag{23}$$

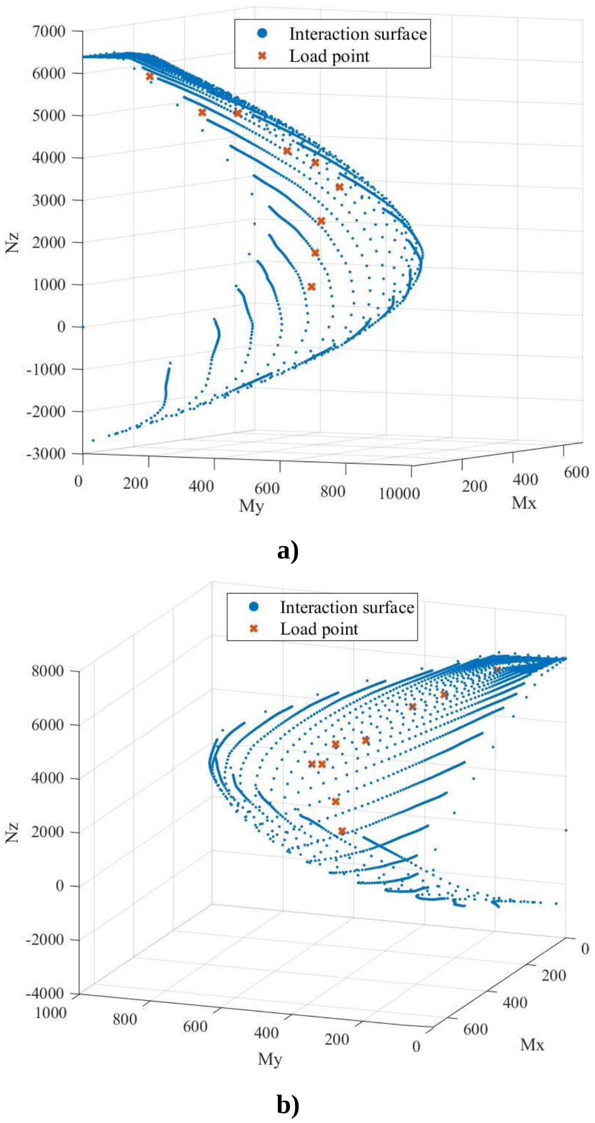


Figure 7. Checking load-bearing capacity using 3D interaction surface

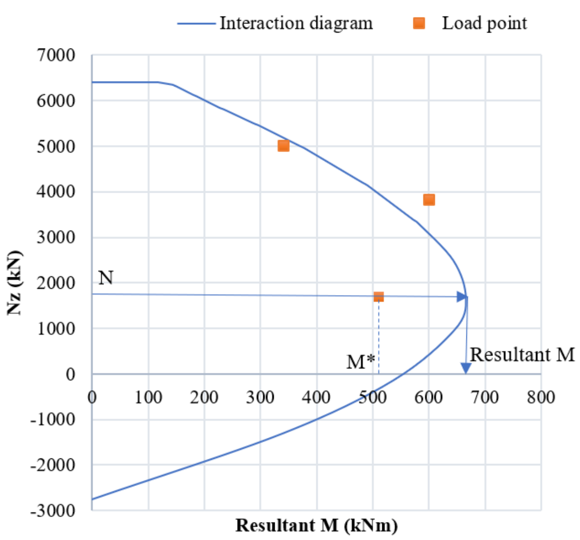


Figure 8. Checking load-bearing capacity using *N-M* interaction diagram-passed load case.

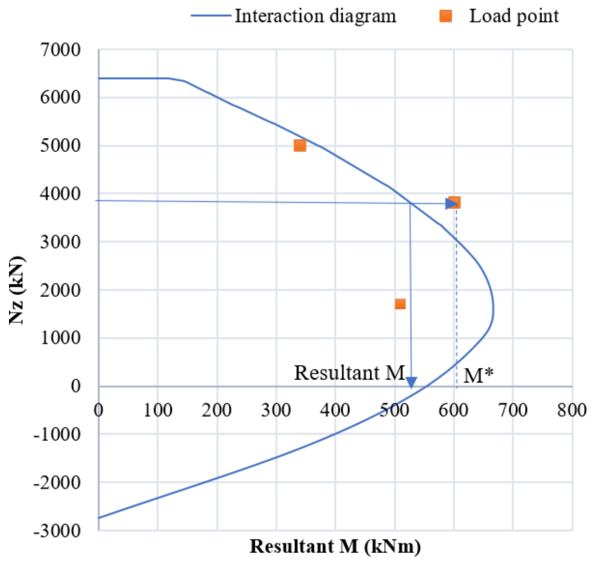


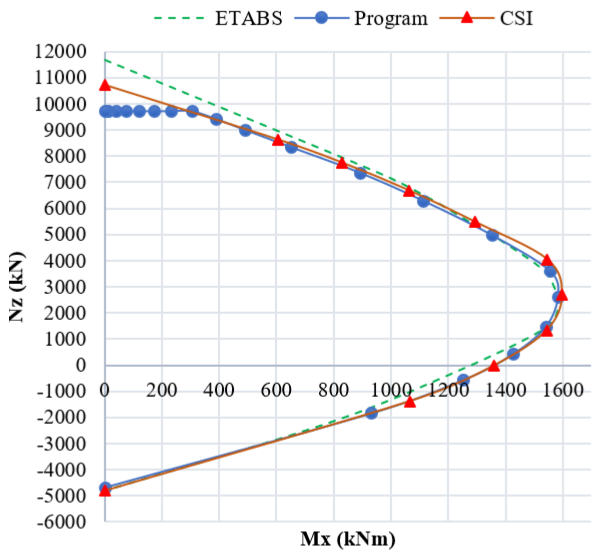
Figure 9. Checking load-bearing capacity using *N-M* interaction diagram-failed load case.

3. MODEL VERIFICATION AND APPLICATION

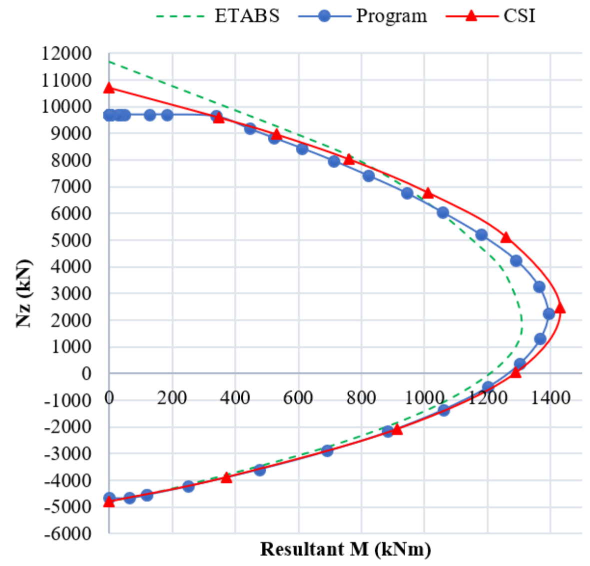
The 3D model generated by the proposed program is verified in comparison to the result from ETABS ver18.1.1 and CSI-Col ver10.0.0.

3.1. Verification for a square column

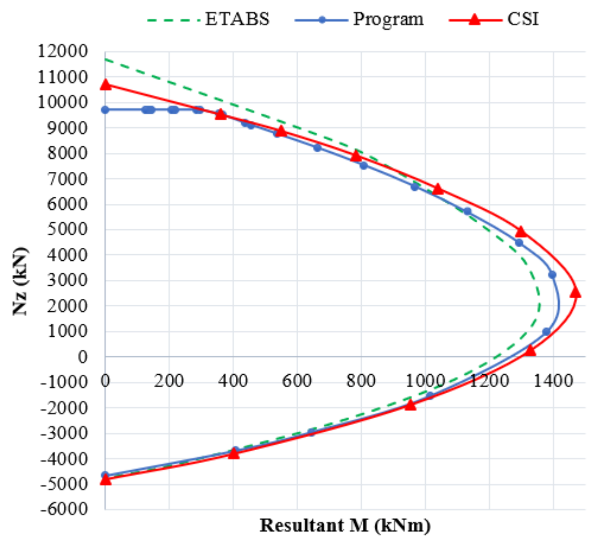
A column has a square cross-section of 70 cm x 70 cm, concrete grade B25, reinforcement CB400-V, 28 Ø 25 mm arranged evenly along the perimeter. The distance between the edge and the central of steel bars a_0 = 4 cm. The length of the column is 7 m. Work condition coefficient of concrete γ_b = 0.85.



a) 0^0



b) 45^0



c) 60^0

Figure 10. *N-M* Interaction diagram of square column

The proposed program, ETABS, and CSI-Col give the same results, with an error of less than 7.2%. There are no noticeable differences in the uniaxial scenario. The maximum error occurs at the balance point, the transition between compression-controlled and tension-controlled points, in case the neutral axis angle is 45^0. The discrepancy can happen because of the method for calculating the force and moment due to the concrete compression area, stress-strain relationships, failure criteria of concrete, and work condition coefficients. In the proposed program, the maximum value of N is 8614 kN due to the coefficient φ, as mentioned in Equation (10).

3.2. Verification for a rectangular column

A column has rectangular cross-section 0.7 m x 0.5 m, concrete grade B25, reinforcement CB400-

V, 16 Ø 25 mm as shown in Figure 8. The distance between the edge and the central of steel bars a_0 = 4 cm. The length of the column is 4.5 m. Work condition coefficient of concrete γ_b = 0.85.

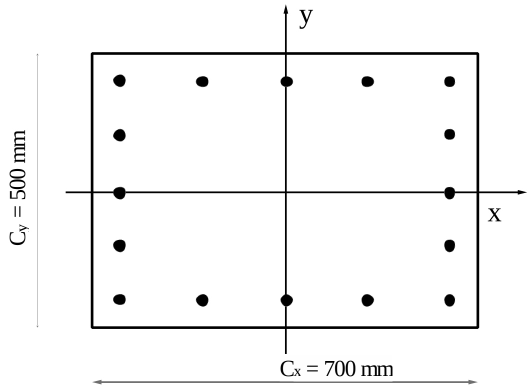
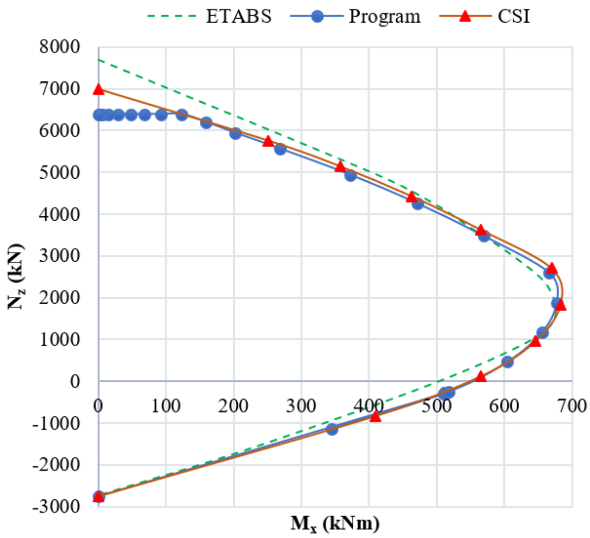
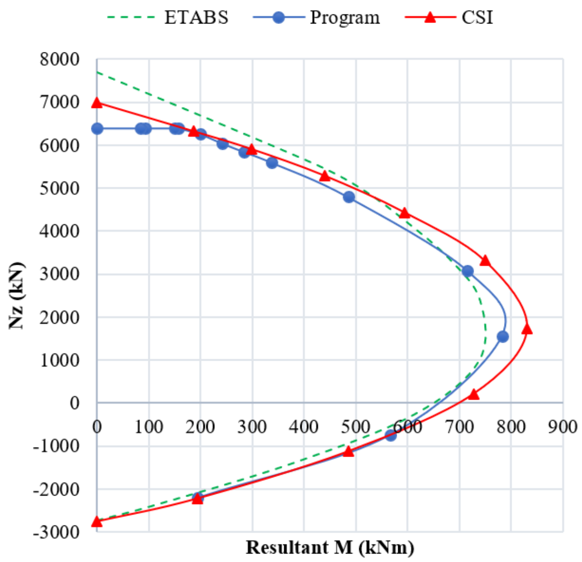


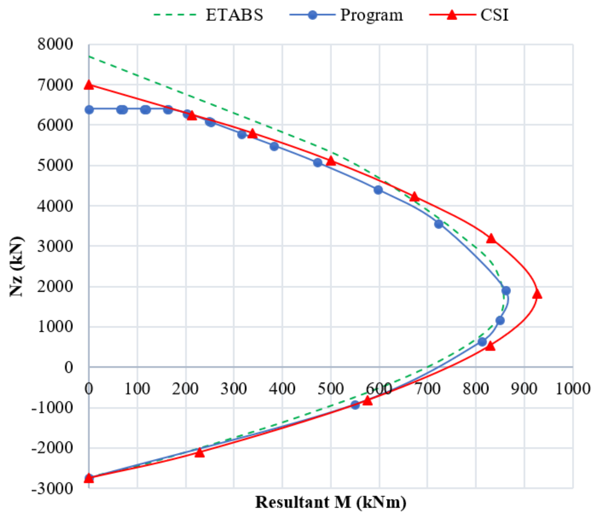
Figure 8. Rectangular column's cross-section



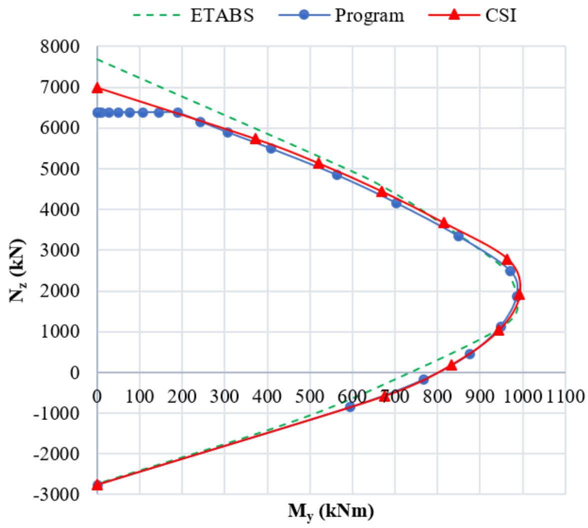
a) 0^0



b) 45^0



c) 60^0



d) 90^0

Figure 11. *N-M* Interaction diagram of rectangular column

Notice: The relationship between the neutral axis angle and the point's coordinates is $\tan\alpha = \frac{M_{ux}}{M_{uy}}\frac{I_y}{I_x}$.

As can be seen from Figure 11, in cases where the neutral axis angle is 0^0 and 90^0, the discrepancies between the three software are negligible. The max error occurs at the balance point in case the neutral axis angle is 60^0 and is less than 7.2%. The maximum axial forces that the column can withstand when considering the buckling factor is 6394.64 kN.

Compared with other research, where the errors are up to 16.8% for square column[13] and 8.74 for rectangular column[15], the proposed program errors are less than 7.2% for both types of cross-section. As a result, the proposed program is reliable. It can be used in practical design, further research and teaching.

3.3. Application for checking load-bearing

capacity

The column is as shown in the subsection 3.2. The internal moments and axial forces from the load combinations are given in Table 1. Internal moments of dead load are $M_{L1x} = 80$ kNm, $M_{L1y} = 300$ kNm.

Question: Checking the load-bearing capacity of the column.

Table 1. Forces and moments from load combinations.

Combo	N (kN)	M_x (kNm)	M_y (kNm)
1	5400	90	315
2	5310	180	360
3	6300	45	180
4	4320	270	450
5	4050	270	540
6	2700	180	630
7	1800	270	540
8	900	315	495
9	3420	315	585

The load points are determined from Equations (11-23), as shown in Table 2.

Table 2. Load points after considering slender column effect

Combo	N (kN)	M^*_x (kNm)	M^*_y (kNm)	M^* (kNm)
1	5400	134.7	326.1	352.8
2	5310	190.3	372.2	418.1
3	6300	162.2	188.4	248.6
4	4320	282.3	462.3	541.7
5	4050	281.6	553.8	621.3
6	2700	185.5	641.6	667.9
7	1800	275.7	547.2	612.8
8	900	319.0	498.9	592.2
9	3420	326.6	598.0	681.4

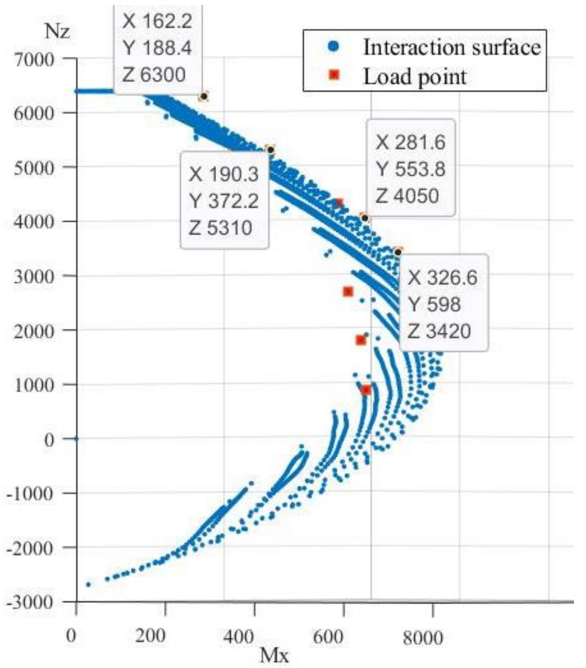


Figure 12. Checking load-bearing capacity using 3D interaction surface

Using 3D interaction surface, the program points out 4 load points outside of the interaction volume, as shown in Figure 12. They come from combination 2, 3, 5 and 9.

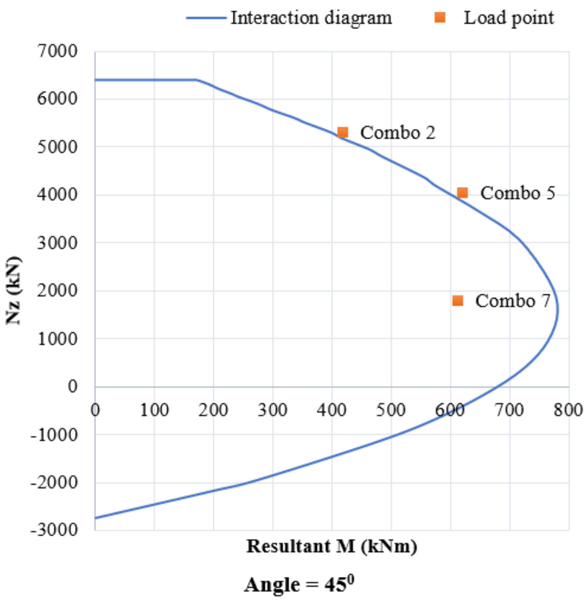


Figure 13. Checking load-bearing capacity using *N-M* interaction diagram (45^0)

Notice: The relationship between the neutral axis angle and the point's coordinates is $\tan\alpha = \frac{M^*_x}{M^*_y}\frac{I_y}{I_x}$

The integrated module in the program to checking the load-bearing capacity using *N-M* curve yields the result in Table 3.

Table 3. Checking load-bearing capacity using *N-M*

interaction diagram

Com	M^* (kNm)	Angle (degree)	Resultant M	Check
1	352.8	51	386.1	Pass
2	418.1	45	395.8	Fail
3	248.6	31	174.3	Fail
4	541.7	40	542.4	Pass
5	621.3	45	594.0	Fail
6	667.9	60	815.7	Pass
7	612.8	45	776.6	Pass
8	592.2	39	720.9	Pass
9	681.4	43	664.9	Fail

As can be seen from Figure 12 and Table 3, the two methods of checking load-bearing capacity show identical results.

4. CONCLUSIONS

This study develops a program to generate 3D interaction surfaces of columns subjected to axial force and biaxial bending. The method is explained in detail, using non-linear deformation models and considering the buckling factor per Vietnamese standard 5574: 2018. The program also includes a module to evaluate the load-bearing capacity of the columns.

The proposed method and program are highly accurate compared to imported software such as ETABS and CSI-Col. The most discrepancy is less than 7.2% for both square and rectangular columns. In addition, 3D views allow the program to check fast the column capacity subjected to various biaxial bending and axial load cases based on TCVN 5574:2018.

A step-by-step procedure to calculate the coefficient of slenderness effect η_x, η_y was introduced. Two simple methods to check the column capacity using 2D interaction diagrams is presented and incorporated into the proposed program. An example of cross-checking the results from 3D views and 2D diagrams was conducted and showed identical checking results.

The proposed program can be used for further research, including investigating the effects of initial conditions on the column capacity, such as uncertainty of materials properties, effective cover, work condition coefficients, stress-strain relationship, etc. However, there is a requirement for validating the program using experiment results.

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