Thiết kế tối ưu bền vững cho thiết kế đường dây truyền tải điện trên không

TÓM TẮT

Hầu hết các phương pháp thiết kế đường dây truyền tải điện trên không trong thực tế hiện nay chủ yếu dựa trên các công thức thực nghiệm. Do đó, bài báo này nhằm đề xuất một quy trình thiết kế tối ưu gồm hai giai đoạn dựa trên thiết kế bền vững để thiết kế đường dây truyền tải điện trên không. Trong giai đoạn thứ nhất, tương ứng với giai đoạn thiết kế thí nghiệm, bốn biến đầu vào gồm loại dây dẫn, kích thước dây dẫn, chiều dài nhịp và lực căng ban đầu; cùng với hai yếu tố nhiễu là nhiệt độ môi trường và tốc độ gió được xem xét, mỗi yếu tố ở ba mức. Ba đáp ứng đầu ra được đánh giá bao gồm: độ võng lớn nhất, tổn thất công suất và chi phí vòng đời. Các yếu tố biến đầu vào và yếu tố nhiễu lần lượt được thiết kế trong ma trận trong và ma trận ngoài theo thiết kế giai thừa toàn phần. Sau đó, các ma trận này được kết hợp để hình thành thiết kế thí nghiệm giai thừa toàn phần dạng mảng chéo L_{81}. Trong giai đoạn thứ hai, các hàm mong muốn được sử dụng để tối thiểu hóa đồng thời ba đáp ứng đầu ra. Cuối cùng, một phân tích so sánh giữa các dạng hàm mong muốn khác nhau được thực hiện, và dựa trên các kết quả thu được, các nghiệm thiết kế tối ưu cho đường dây truyền tải điện trên không được xác định.

Từ khóa: *Thiết kế bền vững, Thiết kế đường dây truyền tải điện trên không, Thiết kế giai thừa toàn phần, Hàm mong muốn.*

Robust design optimization for overhead power transmission line design

ABSTRACT

Most practical design methods for overhead power transmission lines primarily rely on empirical formulas. This paper proposes a robust design–based two-stage optimization framework for overhead power transmission line design. In the first stage, corresponding to the design of experiments or experimental design, four control factors—namely conductor type, conductor size, span length, and pretension—and two noise factors—ambient temperature and wind speed—are considered, each at three levels. Three output responses—maximum sag, power loss, and life cycle cost—are evaluated. The control factors and noise factors are arranged in the inner and outer arrays, respectively, using a full factorial design. These arrays are then combined to form a full factorial L_{81} crossed-array experimental design. In the second stage, desirability functions are employed to simultaneously minimize the three output responses. Finally, a comparative analysis of different desirability formulations is conducted, and based on the obtained results, the optimal design solutions for the overhead transmission line are identified.

Keywords: *Robust design, Overhead power transmission line design, Full factorial design, Desirability functions.*

1. INTRODUCTION

Overhead power transmission lines play a critical role in electric power systems by enabling the transmission and distribution of electrical energy from generation sources to consumption areas. Compared with alternative transmission technologies, overhead lines offer significant advantages in terms of lower investment cost, scalability, and ease of operation and maintenance. Consequently, they remain the predominant solution for both transmission and distribution networks across a wide range of voltage levels. The quality of transmission line design has a direct impact on supply reliability, power losses, operational safety, and the overall economic efficiency of the power system. The design of overhead power transmission lines constitutes a complex engineering problem involving coupled electrical, mechanical, structural, and environmental factors. Key design variables include conductor type and cross-sectional area, span length, pole or tower height, phase configuration, insulation system, as well as conductor sag and tension. These parameters are strongly interdependent rather than independent; variations in a single parameter can simultaneously influence current-carrying capability, electrical clearances, mechanical integrity, and both construction and operational costs. As a result, transmission line design requires a holistic and systematic consideration of multiple interacting constraints. In practical engineering applications, transmission line design is commonly conducted in accordance with technical standards and deterministic calculation methods, with conservative assumptions adopted to ensure safety margins and operational reliability. Although such approaches provide robust designs, they may result in suboptimal solutions characterized by excessive material usage, increased capital expenditure, and higher energy losses. These limitations are further exacerbated by growing transmission demand and increasingly variable environmental conditions. Moreover, uncertainties associated with ambient temperature, wind loading, ice accretion, and long-term material aging introduce additional challenges in both the design process and the accurate assessment of transmission line performance.

Extensive research efforts have been devoted to improving the modeling and analysis of overhead transmission line behavior. In sag–tension and mechanical design, detailed analytical procedures accounting for thermal states, conductor creep, and insulator string characteristics have been developed.[1] Modified ruling span methods have been proposed to calculate sag and tension in multi-span line segments while considering the rotational stiffness of suspension insulator strings.[2] Field measurements and statistical analyses have been used to evaluate discrepancies between designed and actual sag, highlighting the effects of installation practices and modeling uncertainties on design accuracy.[3] Reduced-order, least-squares-based state estimators have also been introduced to estimate sag in leveled span configurations, providing useful tools for mechanical design validation and conductor behavior monitoring.[4] Furthermore, sag–tension relationships under maximum wind loading conditions have been investigated to assess mechanical reliability under extreme weather events.[5] In terms of thermal modeling and ampacity assessment, computational fluid dynamics-based thermal models have demonstrated adequate accuracy in predicting conductor temperature, offering a viable approach for dynamic analysis of distribution networks operating under variable load and weather conditions.[6] The application of high-temperature low-sag conductors and their design implications for capacity enhancement have been evaluated in accordance with international standards such as those issued by CIGRE and IEEE.[7] Transmission line ratings based on the relationship between conductor temperature and sag have been determined using the ruling span principle and tension monitoring systems.[8] For design validation and condition monitoring, predictive sag–load models incorporating temperature effects and mechanical deformation have been proposed,[9] while vibration-based tension estimation techniques using accelerometer measurements have been introduced to support mechanical monitoring.[10] In addition, non-contact electric field sensing methods have been applied to transmission line sag measurement and simulation.[11] Advanced conductor technologies, including ACCC, ACSS, and ACCR conductors, have been compared in terms of ampacity and sag performance using reconductoring approaches relevant to design optimization.[12] A comprehensive review of conductor design parameters and associated monitoring technologies is provided.[13] Despite these advances, conventional transmission line design practice continues to rely largely on deterministic procedures and empirically derived design rules. In recent years, optimization-based approaches have attracted increasing attention as effective tools for improving transmission line planning and design. Cost assessment and optimization models employing genetic algorithms have been proposed to minimize the total cost of overhead transmission lines while satisfying design constraints.[14] Integrated optimization frameworks for route selection and tower placement using graph algorithms and dynamic programming techniques have also been developed.[15] Heuristic optimization methods have been applied to optimize conductor bundle geometry with the objective of enhancing transmission capacity while reducing overall cost.[16] Multi-objective optimization approaches considering trade-offs between electrical and mechanical performance have been investigated using genetic algorithms.[17] More recently, metaheuristic techniques such as the whale optimization algorithm have been employed to estimate optimal parameters of overhead AC transmission lines, enabling improved tuning of design variables under electrical constraints.[18]

Although substantial progress has been made in the modeling, analysis, and optimization of overhead power transmission lines, existing studies predominantly emphasize performance improvement under nominal operating conditions. Mechanical and thermal analyses typically focus on accurate prediction of sag, tension, and ampacity, while optimization-based approaches aim to minimize cost or maximize capacity through advanced search algorithms. However, these studies often assume fixed design parameters and operating environments, thereby limiting their applicability in practical scenarios where uncertainties are unavoidable.

Moreover, existing studies have not yet proposed a unified framework for optimal design calculation of overhead power transmission lines that can be generally applied to different line configurations. Factors such as ambient temperature and wind speed, which can significantly influence transmission line design, are often not explicitly considered in the optimal design process. In addressing the technical optimization problem, the robust design methodology proposed by Taguchi can be employed as an effective solution and may be regarded as an alternative to conventional optimization techniques. In quality engineering, products or processes are commonly modeled as a box or system characterized by inputs and outputs. Taguchi classifies the input variables into two categories: control factors (denoted by x) and noise factors (denoted by z). Taguchi proposed a two-step procedure for solving optimal design problems, consisting of the construction of a design of experiments and the use of signal-to-noise ratio (SNR) analysis to identify optimal solutions. In the design of experiments stage, Taguchi introduced a set of orthogonal arrays instead of traditional experimental design methods. Among more than

60 types of SNRs, three have been most widely adopted: nominal-is-best, larger-is-better, and smaller-is-better. However, the orthogonal arrays and SNR formulations proposed by Taguchi have been criticized by statisticians and researchers in the literature. [19–21]

Based on the Taguchi philosophy, this paper develops a general multi-objective optimization framework for the design of overhead power transmission lines. In the experimental design stage, a general full factorial design is employed with four control factors—namely conductor type, conductor size, span length, and pretension—each considered at three levels, and two noise factors—ambient temperature and wind speed—also considered at three levels. Three output responses are evaluated: maximum sag, power loss, and life cycle cost (LCC). With the objective of minimizing these three responses, several desirability functions are applied to identify optimal solutions. The results obtained from different desirability functions are compared to identify the best solutions. To the best of our knowledge, this is the first attempt to apply the Taguchi philosophy in this area. An overview of the proposed robust design-based two-stage optimization framework for overhead transmission line design is shown in Figure 1.

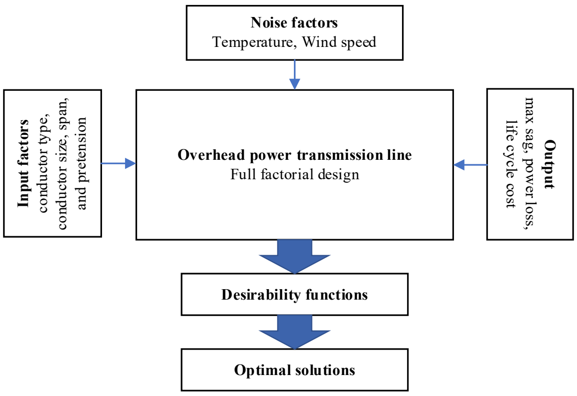

Figure 1. Proposed robust design-based two-stage optimization framework for overhead transmission line design.

2. PROPOSED ROBUST DESIGN OPTIMIZATION

This section presents the proposed robust design–based two-stage optimization framework for overhead power transmission line design. The methodology integrates the Taguchi robust design philosophy with an advanced desirability function-based optimization algorithm to achieve design solutions that are not only optimal under nominal conditions but also robust against environmental and operational uncertainties.

2.1. Design of experiments stage

The overhead transmission line design problem is formulated as a multi-input–multi-output engineering system, in which a set of controllable design variables and uncontrollable noise factors jointly influence the performance of the transmission line. The objective is to determine the optimal combination of design parameters that minimizes key performance indices while maintaining robustness under varying environmental conditions.

Unlike the classical Taguchi approach that relies on predefined orthogonal arrays, this study employs a full factorial experimental design to comprehensively capture interactions among control and noise factors. This choice ensures accurate modeling of complex interdependencies between electrical and mechanical parameters that are common in overhead transmission line systems. For each experimental run, the transmission line responses are evaluated under all combinations of noise factor levels, allowing the influence of environmental variability on system performance to be explicitly quantified.

Based on engineering practice, four control factors reflecting realistic engineering choices are selected: conductor type (x_1), conductor size (x_2), span (x_3), and pretension (x_4). To account for environmental uncertainty, two noise factors, representing typical, adverse, and extreme operating conditions, are incorporated: ambient temperature (z_1) and wind speed (z_2).

Three output responses are selected to evaluate transmission line performance:

- ✓ Maximum conductor sag (y_1), which directly affects electrical clearance and mechanical safety;
- ✓ Power loss (y_2), representing the electrical efficiency of the line;
- ✓ LCC (y_3), which includes material, construction, operation, and maintenance costs.

The four control factors were selected based on standard overhead transmission line design practice, as they are directly controllable by designers and have a dominant influence on both electrical and mechanical performance. The two noise factors represent key environmental uncertainties that cannot be controlled in operation but significantly affect sag, resistance, and power loss. Each factor was defined at three levels to represent low, nominal, and high operating conditions, allowing nonlinear trends and robustness effects to be captured while maintaining a manageable experimental size.

2.1.1. Maximum conductor sag

For typical overhead transmission spans (≤ 500 m), the conductor is modeled as a flexible cable subjected to uniformly distributed loading. Under normal operating conditions, the parabolic approximation is widely accepted and yields sufficiently accurate results for design and optimization studies.

The mid-span sag is expressed as:

$$f = \frac{\omega L^2}{8H} \tag{1}$$

where f is mid-span sag (m), ω is resultant weight per unit length (N/m), L is span length (m), and H is horizontal component of conductor tension (N).

The weight per unit length is given by:

$$\omega = mg \tag{2}$$

where m is the conductor mass per unit length (kg/m) and g is the gravitational acceleration (9.81 m/s^2).

The horizontal tension is determined by the pretension ratio:

$$H = \eta H_{rated} \tag{3}$$

where η denotes the pretension level and H_{rated} is the rated horizontal tension of the conductor.

To account for environmental uncertainties such as temperature variation and wind speed, the maximum sag is corrected as

$$f_{\max} = \frac{\omega L^2}{8H}\left[1 + \alpha_s\left(T - T_0\right) + \beta_\omega V_\omega\right] \tag{4}$$

where T is ambient temperature (^{0}C), T_0 is reference temperature (typically 20^{0}C), V_ω is wind speed (m/s), and α_s, β_ω are temperature and wind correction coefficients.

The parabolic approximation employed in this study assumes relatively small sag-to-span ratios and uniformly distributed loading, which is generally valid for typical transmission spans under normal operating conditions. For very long spans or extreme weather events involving high wind speeds, ice accretion, or large temperature gradients, catenary effects and nonlinear geometric behavior may become significant. In such cases, the simplified sag model may introduce deviations, and more advanced catenary-based or finite-element models should be used in detailed design stages.

2.1.2. Power loss

The active power loss in an overhead conductor is governed by Joule heating and is expressed as:

$$P_{loss} = I^2 R\left(T\right) \tag{5}$$

where I is the line current (A) and $R(T)$ is the conductor resistance at operating temperature T.

The temperature-dependent resistance is modeled as:

$$R\left(T\right) = R_{20}\left[1 + \alpha_{20}\left(T - 20\right)\right] \tag{6}$$

where R_{20} is the resistance at 20^{0}C and α_R is the temperature coefficient of resistance. For a given span length L, the total power loss is:

$$P_{loss,span} = I^2 R\left(T\right)\frac{L}{1000}. \tag{7}$$

2.1.3. Life cycle cost

The LCC of an overhead transmission line is defined as the sum of capital investment, energy loss cost, and maintenance cost over the service lifetime.[22]

$$LCC = C_{capital} + C_{loss} + C_{main} \tag{8}$$

The capital cost is primarily determined by the conductor material and size:

$$C_{capital} = C_{cond}\left(x_1, x_2\right). \tag{9}$$

The cumulative energy loss cost over the service life is calculated as:

$$C_{loss} = P_{loss} h_{life} c_{energy} \tag{10}$$

where h_{life} denotes the total operating hours over the service life and c_{energy} is the unit cost of electrical energy.

Mechanical stress and increased sag result in higher inspection and maintenance requirements. A linear sag-dependent maintenance cost model is adopted:

$$C_{main} = k_m f_{\max} \tag{11}$$

where k_m is maintenance cost coefficient.

A linear sag–maintenance cost relationship is adopted to maintain model simplicity and computational efficiency. Increased sag typically leads to higher inspection and clearance management efforts, for which linear approximations are widely used in preliminary economic analyses. The model is therefore intended for comparative design assessment within normal operating sag ranges.

The LCC formulation is a simplified model for robust, preliminary design optimization and does not explicitly account for stochastic failures, aging, or condition-based maintenance. The framework therefore supports early-stage decision-making under uncertainty rather than detailed reliability or asset management analyses.

2.2. Optimization stage

The design problem is therefore formulated as a multi-objective minimization problem:

$$\min\{y_1(\mathbf{x},\mathbf{z}), y_2(\mathbf{x},\mathbf{z}), y_3(\mathbf{x},\mathbf{z})\} \qquad (12)$$

subject to $\mathbf{x}^{\min} \le \mathbf{x} \le \mathbf{x}^{\max}$

where $\mathbf{x}$ represents the vector of control factors (conductor type, conductor size, span, pretension).

Three different types of desirability functions for three different cases are as follows:

If the target is the best, the desirability function is

$$d_i(y_i) = \begin{cases} 0 & \text{if } y_i < L_i \\ \left(\dfrac{y_i - L_i}{T_i - L_i}\right)^s & \text{if } L_i \le y_i \le T_i \\ \left(\dfrac{y_i - U_i}{T_i - U_i}\right)^t & \text{if } T_i \le y_i \le U_i \\ 0 & \text{if } y_i > U_i \end{cases} \qquad (13)$$

If the maximize is better, the desirability function is

$$d_i(y_i) = \begin{cases} 0 & \text{if } y_i < L_i \\ \left(\dfrac{y_i - L_i}{T_i - L_i}\right)^s & \text{if } L_i \le y_i \le T_i \\ 1 & \text{if } y_i > T_i \end{cases} \qquad (14)$$

If the minimize is better, the desirability function is

$$d_i(y_i) = \begin{cases} 1 & \text{if } y_i < T_i \\ \left(\dfrac{y_i - U_i}{T_i - U_i}\right)^t & \text{if } T_i \le y_i \le U_i \\ 0 & \text{if } y_i > U_i \end{cases} \qquad (15)$$

where L_i, T_i, and U_i are the lower, target, and upper values of the i^{th} function, respectively. r, s, and t are the exponents of each desirability function. The lower, target, and upper values of the desirability functions were determined based on acceptable engineering limits, relevant design standards, and the observed response ranges obtained from the experimental design. The exponents (r, s, t) were selected to reflect the relative importance of each performance objective and the severity of penalties for deviations from desired values. To assess the influence of desirability formulation, different desirability functions were compared, providing insight into the sensitivity of the optimal solution to these parameters.

The overall desirability function is shown as

$$D = \left[d_1(y_1) d_2(y_2) \ldots d_k(y_k)\right]^{1/k} \qquad (16)$$

where k is the number of responses.

3. CASE STUDY

In this study, a 22-kV overhead power transmission line is selected as the case study to demonstrate the proposed optimization framework. For the experimental design, each control factor and each noise factor is defined at three discrete and practically meaningful levels, enabling a systematic evaluation of their effects on the transmission line performance as:

- ✓ Conductor type (x_1): ACSR, AAAC, and ACCC (coded value: 1, 2, 3);
- ✓ Conductor size (x_2): 95, 150, and 240 (m^2) (coded value: 1, 2, 3);
- ✓ Span (x_3): 100, 150, and 200 (m) (coded value: 1, 2, 3);
- ✓ Pretension (x_4): 0.3, 0.5, and 0.7 (of rated) (coded value: 1, 2, 3).

Similarly, each noise factor is defined at three discrete levels to reflect realistic variations in environmental and operating conditions encountered in practice, as summarized as follows:

- ✓ Ambient temperature (z_1): – 10, 20, and 40 (^{0}C) (coded value: 1, 2, 3);

- ✓ Wind speed (z_2): 0, 15, and 30 (m/s) (coded value: 1, 2, 3).

The parameters of the ACSR, AAAC, and ACCC conductors with different sizes are summarized in Tables 1, 2, and 3, respectively, including their electrical, mechanical, and physical characteristics relevant to transmission line design and performance evaluation.

Table 1. Parameters of conductor ACSR.

Size (mm^2)	Mass (kg/m)	R_{20} (Ω/km)	Cost (USD/km)
95	0.38	0.193	8000
150	0.62	0.126	12000
240	0.98	0.078	18000

Table 2. Parameters of conductor AAAC.

Size (mm^2)	Mass (kg/m)	R_{20} (Ω/km)	Cost (USD/km)
95	0.35	0.195	8500
150	0.58	0.127	12500
240	0.92	0.079	18500

Table 3. Parameters of conductor ACCC.

Size (mm^2)	Mass (kg/m)	R_{20} (Ω/km)	Cost (USD/km)
95	0.32	0.50	11000
150	0.50	0.125	15000
240	0.82	0.077	23000

Moreover, the additional parameters used in the overhead power transmission line design are as follows: the rated line current is 300 A; the gravitational acceleration is 9.81 m/s^2; the rated horizontal tension is 50000 N; the thermal sag coefficient is 1.2×10^{-5} $^0C^{-1}$; the wind sag coefficient is 0.005 (m/s)$^{-1}$; the resistance temperature coefficient is 0.004^0C^{-1}, the service life hours is 30 years, corresponding to 262800 h; the energy price is 0.10 USD/kWh, and the maintenance cost factor is 1000 USD/m.

Based on the selected control and noise factors and their associated coded levels, a full factorial L_{81} crossed-array experimental design is constructed for the three output responses, as summarized in Tables 4, 5, and 6. In this crossed-array structure, the inner array represents all possible combinations of the control factors, while the outer array accounts for the combinations of the noise factors, enabling a systematic evaluation of the robustness of the design against environmental and operational variations. This design framework facilitates the simultaneous assessment of factor interactions and variability effects on the system performance.

Table 4. Full factorial design L_{81} array for maximum conductor sag.

					Runs	Outer array								
						1	2	3	4	5	6	7	8	9
					z_1	1	2	3	1	2	3	1	2	3
					z_2	1	1	1	2	2	2	3	3	3
Runs	Inner array					Maximum conductor sag (m)								
	x_1	x_2	x_3	x_4		y_{11}	y_{12}	y_{13}	y_{14}	y_{15}	y_{16}	y_{17}	y_{18}	y_{19}
1	1	1	1	1		0.3105	0.3107	0.3107	0.3338	0.3339	0.3340	0.3571	0.3572	0.3573
2	1	1	1	2		0.1863	0.1864	0.1864	0.2003	0.2004	0.2004	0.2143	0.2143	0.2144
3	1	1	1	3		0.1331	0.1331	0.1332	0.1431	0.1431	0.1432	0.1531	0.1531	0.1531
4	1	1	2	1		0.6987	0.6990	0.6991	0.7511	0.7514	0.7516	0.8036	0.8038	0.8040

5	1	1	2	2	0.4192	0.4194	0.4195	0.4507	0.4508	0.4509	0.4821	0.4823	0.4824
6	1	1	2	3	0.2994	0.2996	0.2996	0.3219	0.3220	0.3221	0.3444	0.3445	0.3446
7	1	1	3	1	1.2422	1.2426	1.2429	1.3353	1.3358	1.3361	1.4285	1.4290	1.4293
8	1	1	3	2	0.7453	0.7456	0.7457	0.8012	0.8015	0.8017	0.8571	0.8574	0.8576
9	1	1	3	3	0.5324	0.5325	0.5327	0.5723	0.5725	0.5726	0.6122	0.6124	0.6126
10	1	2	1	1	0.5067	0.5069	0.5070	0.5447	0.5449	0.5450	0.5827	0.5829	0.5830
11	1	2	1	2	0.3040	0.3041	0.3042	0.3268	0.3269	0.3270	0.3496	0.3497	0.3498
12	1	2	1	3	0.2171	0.2172	0.2173	0.2334	0.2335	0.2336	0.2497	0.2498	0.2499
13	1	2	2	1	1.1400	1.1404	1.1407	1.2255	1.2259	1.2262	1.3111	1.3115	1.3117
14	1	2	2	2	0.6840	0.6842	0.6844	0.7353	0.7356	0.7357	0.7866	0.7869	0.7870
15	1	2	2	3	0.4886	0.4887	0.4889	0.5252	0.5254	0.5255	0.5619	0.5621	0.5622
16	1	2	3	1	2.0267	2.0274	2.0279	2.1787	2.1795	2.1799	2.3308	2.3315	2.3320
17	1	2	3	2	1.2160	1.2164	1.2167	1.3072	1.3077	1.3080	1.3985	1.3989	1.3992
18	1	2	3	3	0.8686	0.8689	0.8691	0.9337	0.9341	0.9343	0.9989	0.9992	0.9994
19	1	3	1	1	0.8009	0.8012	0.8013	0.8609	0.8612	0.8614	0.9210	0.9213	0.9215
20	1	3	1	2	0.4805	0.4807	0.4808	0.5166	0.5167	0.5169	0.5526	0.5528	0.5529
21	1	3	1	3	0.3432	0.3434	0.3434	0.3690	0.3691	0.3692	0.3947	0.3949	0.3949
22	1	3	2	1	1.8019	1.8026	1.8030	1.9371	1.9378	1.9382	2.0723	2.0730	2.0734
23	1	3	2	2	1.0812	1.0816	1.0818	1.1623	1.1627	1.1629	1.2434	1.2438	1.2440
24	1	3	2	3	0.7723	0.7725	0.7727	0.8302	0.8305	0.8307	0.8881	0.8884	0.8886
25	1	3	3	1	3.2034	3.2046	3.2054	3.4438	3.4449	3.4457	3.6841	3.6853	3.6861
26	1	3	3	2	1.9221	1.9228	1.9232	2.0663	2.0670	2.0674	2.2105	2.2112	2.2116
27	1	3	3	3	1.3729	1.3734	1.3737	1.4759	1.4764	1.4767	1.5789	1.5794	1.5797
28	2	1	1	1	0.2860	0.2861	0.2862	0.3075	0.3076	0.3077	0.3289	0.3290	0.3291
29	2	1	1	2	0.1716	0.1717	0.1717	0.1845	0.1846	0.1846	0.1974	0.1974	0.1975
30	2	1	1	3	0.1226	0.1226	0.1227	0.1318	0.1318	0.1319	0.1410	0.1410	0.1410
31	2	1	2	1	0.6435	0.6438	0.6439	0.6918	0.6921	0.6922	0.7401	0.7403	0.7405
32	2	1	2	2	0.3861	0.3863	0.3864	0.4151	0.4152	0.4153	0.4441	0.4442	0.4443

33	2	1	2	3
34	2	1	3	1
35	2	1	3	2
36	2	1	3	3
37	2	2	1	1
38	2	2	1	2
39	2	2	1	3
40	2	2	2	1
41	2	2	2	2
42	2	2	2	3
43	2	2	3	1
44	2	2	3	2
45	2	2	3	3
46	2	3	1	1
47	2	3	1	2
48	2	3	1	3
49	2	3	2	1
50	2	3	2	2
51	2	3	2	3
52	2	3	3	1
53	2	3	3	2
54	2	3	3	3
55	3	1	1	1
56	3	1	1	2
57	3	1	1	3
58	3	1	2	1
59	3	1	2	2
60	3	1	2	3

0.2758	0.2759	0.2760	0.2965	0.2966	0.2967	0.3172	0.3173	0.3174
1.1441	1.1445	1.1448	1.2299	1.2303	1.2306	1.3158	1.3162	1.3164
0.6865	0.6867	0.6869	0.7380	0.7382	0.7384	0.7895	0.7897	0.7899
0.4903	0.4905	0.4906	0.5271	0.5273	0.5274	0.5639	0.5641	0.5642
0.4740	0.4742	0.4743	0.5095	0.5097	0.5098	0.5451	0.5453	0.5454
0.2844	0.2845	0.2846	0.3057	0.3058	0.3059	0.3271	0.3272	0.3272
0.2031	0.2032	0.2033	0.2184	0.2184	0.2185	0.2336	0.2337	0.2337
1.0665	1.0668	1.0671	1.1465	1.1469	1.1471	1.2265	1.2269	1.2271
0.6399	0.6401	0.6403	0.6879	0.6881	0.6883	0.7359	0.7361	0.7363
0.4571	0.4572	0.4573	0.4913	0.4915	0.4916	0.5256	0.5258	0.5259
1.8959	1.8966	1.8971	2.0382	2.0388	2.0393	2.1804	2.1811	2.1815
1.1376	1.1380	1.1382	1.2229	1.2233	1.2236	1.3082	1.3087	1.3089
0.8125	0.8128	0.8130	0.8735	0.8738	0.8740	0.9345	0.9348	0.9349
0.7518	0.7521	0.7523	0.8082	0.8085	0.8087	0.8646	0.8649	0.8651
0.4511	0.4513	0.4514	0.4849	0.4851	0.4852	0.5188	0.5189	0.5191
0.3222	0.3223	0.3224	0.3464	0.3465	0.3466	0.3706	0.3707	0.3708
1.6916	1.6922	1.6926	1.8185	1.8191	1.8195	1.9454	1.9461	1.9465
1.0150	1.0153	1.0156	1.0911	1.0915	1.0917	1.1673	1.1676	1.1679
0.7250	0.7252	0.7254	0.7794	0.7796	0.7798	0.8338	0.8340	0.8342
3.0073	3.0084	3.0091	3.2329	3.2340	3.2348	3.4586	3.4597	3.4604
1.8044	1.8050	1.8055	1.9398	1.9404	1.9409	2.0751	2.0758	2.0762
1.2889	1.2893	1.2896	1.3855	1.3860	1.3863	1.4822	1.4827	1.4830
0.2615	0.2616	0.2617	0.2811	0.2812	0.2813	0.3007	0.3008	0.3009
0.1569	0.1570	0.1570	0.1687	0.1687	0.1688	0.1804	0.1805	0.1805
0.1121	0.1121	0.1121	0.1205	0.1205	0.1205	0.1289	0.1289	0.1290
0.5884	0.5886	0.5887	0.6325	0.6327	0.6329	0.6767	0.6769	0.6770
0.3530	0.3532	0.3532	0.3795	0.3796	0.3797	0.4060	0.4061	0.4062
0.2522	0.2523	0.2523	0.2711	0.2712	0.2712	0.2900	0.2901	0.2902

61	3	1	3	1		1.0460	1.0464	1.0467	1.1245	1.1249	1.1251	1.2030	1.2034	1.2036
62	3	1	3	2		0.6276	0.6278	0.6280	0.6747	0.6749	0.6751	0.7218	0.7220	0.7222
63	3	1	3	3		0.4483	0.4485	0.4486	0.4819	0.4821	0.4822	0.5156	0.5157	0.5158
64	3	2	1	1		0.4086	0.4088	0.4088	0.4393	0.4394	0.4395	0.4699	0.4701	0.4702
65	3	2	1	2		0.2452	0.2453	0.2453	0.2636	0.2636	0.2637	0.2819	0.2820	0.2821
66	3	2	1	3		0.1751	0.1752	0.1752	0.1883	0.1883	0.1884	0.2014	0.2015	0.2015
67	3	2	2	1		0.9194	0.9197	0.9199	0.9883	0.9887	0.9889	1.0573	1.0576	1.0579
68	3	2	2	2		0.5516	0.5518	0.5519	0.5930	0.5932	0.5933	0.6344	0.6346	0.6347
69	3	2	2	3		0.3940	0.3942	0.3942	0.4236	0.4237	0.4238	0.4531	0.4533	0.4534
70	3	2	3	1		1.6344	1.6350	1.6354	1.7570	1.7576	1.7580	1.8797	1.8803	1.8806
71	3	2	3	2		0.9806	0.9810	0.9812	1.0542	1.0546	1.0548	1.1278	1.1282	1.1284
72	3	2	3	3		0.7005	0.7007	0.7009	0.7530	0.7533	0.7534	0.8056	0.8058	0.8060
73	3	3	1	1		0.6701	0.6704	0.6705	0.7204	0.7206	0.7208	0.7707	0.7709	0.7711
74	3	3	1	2		0.4021	0.4022	0.4023	0.4322	0.4324	0.4325	0.4624	0.4625	0.4626
75	3	3	1	3		0.2872	0.2873	0.2874	0.3087	0.3088	0.3089	0.3303	0.3304	0.3305
76	3	3	2	1		1.5077	1.5083	1.5086	1.6209	1.6214	1.6218	1.7340	1.7345	1.7349
77	3	3	2	2		0.9046	0.9050	0.9052	0.9725	0.9728	0.9731	1.0404	1.0407	1.0409
78	3	3	2	3		0.6462	0.6464	0.6466	0.6947	0.6949	0.6950	0.7431	0.7434	0.7435
79	3	3	3	1		2.6804	2.6814	2.6820	2.8815	2.8825	2.8831	3.0826	3.0836	3.0843
80	3	3	3	2		1.6083	1.6088	1.6092	1.7289	1.7295	1.7299	1.8496	1.8502	1.8506
81	3	3	3	3		1.1488	1.1492	1.1494	1.2349	1.2354	1.2356	1.3211	1.3215	1.3218

Table 5. Full factorial design L_{81} array for power loss.

Runs	Power loss (kW)								
	y_{21}	y_{22}	y_{23}	y_{24}	y_{25}	y_{26}	y_{27}	y_{28}	y_{29}
1	1.5286	1.7370	1.8760	1.5286	1.7370	1.8760	1.5286	1.7370	1.8760
2	1.5286	1.7370	1.8760	1.5286	1.7370	1.8760	1.5286	1.7370	1.8760
3	1.5286	1.7370	1.8760	1.5286	1.7370	1.8760	1.5286	1.7370	1.8760
4	2.2928	2.6055	2.8139	2.2928	2.6055	2.8139	2.2928	2.6055	2.8139

5	2.2928	2.6055	2.8139	2.2928	2.6055	2.8139	2.2928	2.6055	2.8139
6	2.2928	2.6055	2.8139	2.2928	2.6055	2.8139	2.2928	2.6055	2.8139
7	3.0571	3.4740	3.7519	3.0571	3.4740	3.7519	3.0571	3.4740	3.7519
8	3.0571	3.4740	3.7519	3.0571	3.4740	3.7519	3.0571	3.4740	3.7519
9	3.0571	3.4740	3.7519	3.0571	3.4740	3.7519	3.0571	3.4740	3.7519
10	0.9979	1.1340	1.2247	0.9979	1.1340	1.2247	0.9979	1.1340	1.2247
11	0.9979	1.1340	1.2247	0.9979	1.1340	1.2247	0.9979	1.1340	1.2247
12	0.9979	1.1340	1.2247	0.9979	1.1340	1.2247	0.9979	1.1340	1.2247
13	1.4969	1.7010	1.8371	1.4969	1.7010	1.8371	1.4969	1.7010	1.8371
14	1.4969	1.7010	1.8371	1.4969	1.7010	1.8371	1.4969	1.7010	1.8371
15	1.4969	1.7010	1.8371	1.4969	1.7010	1.8371	1.4969	1.7010	1.8371
16	1.9958	2.2680	2.4494	1.9958	2.2680	2.4494	1.9958	2.2680	2.4494
17	1.9958	2.2680	2.4494	1.9958	2.2680	2.4494	1.9958	2.2680	2.4494
18	1.9958	2.2680	2.4494	1.9958	2.2680	2.4494	1.9958	2.2680	2.4494
19	0.6178	0.7020	0.7582	0.6178	0.7020	0.7582	0.6178	0.7020	0.7582
20	0.6178	0.7020	0.7582	0.6178	0.7020	0.7582	0.6178	0.7020	0.7582
21	0.6178	0.7020	0.7582	0.6178	0.7020	0.7582	0.6178	0.7020	0.7582
22	0.9266	1.0530	1.1372	0.9266	1.0530	1.1372	0.9266	1.0530	1.1372
23	0.9266	1.0530	1.1372	0.9266	1.0530	1.1372	0.9266	1.0530	1.1372
24	0.9266	1.0530	1.1372	0.9266	1.0530	1.1372	0.9266	1.0530	1.1372
25	1.2355	1.4040	1.5163	1.2355	1.4040	1.5163	1.2355	1.4040	1.5163
26	1.2355	1.4040	1.5163	1.2355	1.4040	1.5163	1.2355	1.4040	1.5163
27	1.2355	1.4040	1.5163	1.2355	1.4040	1.5163	1.2355	1.4040	1.5163
28	1.5444	1.7550	1.8954	1.5444	1.7550	1.8954	1.5444	1.7550	1.8954
29	1.5444	1.7550	1.8954	1.5444	1.7550	1.8954	1.5444	1.7550	1.8954
30	1.5444	1.7550	1.8954	1.5444	1.7550	1.8954	1.5444	1.7550	1.8954
31	2.3166	2.6325	2.8431	2.3166	2.6325	2.8431	2.3166	2.6325	2.8431
32	2.3166	2.6325	2.8431	2.3166	2.6325	2.8431	2.3166	2.6325	2.8431

33	2.3166	2.6325	2.8431	2.3166	2.6325	2.8431	2.3166	2.6325	2.8431
34	3.0888	3.5100	3.7908	3.0888	3.5100	3.7908	3.0888	3.5100	3.7908
35	3.0888	3.5100	3.7908	3.0888	3.5100	3.7908	3.0888	3.5100	3.7908
36	3.0888	3.5100	3.7908	3.0888	3.5100	3.7908	3.0888	3.5100	3.7908
37	1.0058	1.1430	1.2344	1.0058	1.1430	1.2344	1.0058	1.1430	1.2344
38	1.0058	1.1430	1.2344	1.0058	1.1430	1.2344	1.0058	1.1430	1.2344
39	1.0058	1.1430	1.2344	1.0058	1.1430	1.2344	1.0058	1.1430	1.2344
40	1.5088	1.7145	1.8517	1.5088	1.7145	1.8517	1.5088	1.7145	1.8517
41	1.5088	1.7145	1.8517	1.5088	1.7145	1.8517	1.5088	1.7145	1.8517
42	1.5088	1.7145	1.8517	1.5088	1.7145	1.8517	1.5088	1.7145	1.8517
43	2.0117	2.2860	2.4689	2.0117	2.2860	2.4689	2.0117	2.2860	2.4689
44	2.0117	2.2860	2.4689	2.0117	2.2860	2.4689	2.0117	2.2860	2.4689
45	2.0117	2.2860	2.4689	2.0117	2.2860	2.4689	2.0117	2.2860	2.4689
46	0.6257	0.7110	0.7679	0.6257	0.7110	0.7679	0.6257	0.7110	0.7679
47	0.6257	0.7110	0.7679	0.6257	0.7110	0.7679	0.6257	0.7110	0.7679
48	0.6257	0.7110	0.7679	0.6257	0.7110	0.7679	0.6257	0.7110	0.7679
49	0.9385	1.0665	1.1518	0.9385	1.0665	1.1518	0.9385	1.0665	1.1518
50	0.9385	1.0665	1.1518	0.9385	1.0665	1.1518	0.9385	1.0665	1.1518
51	0.9385	1.0665	1.1518	0.9385	1.0665	1.1518	0.9385	1.0665	1.1518
52	1.2514	1.4220	1.5358	1.2514	1.4220	1.5358	1.2514	1.4220	1.5358
53	1.2514	1.4220	1.5358	1.2514	1.4220	1.5358	1.2514	1.4220	1.5358
54	1.2514	1.4220	1.5358	1.2514	1.4220	1.5358	1.2514	1.4220	1.5358
55	1.5048	1.7100	1.8468	1.5048	1.7100	1.8468	1.5048	1.7100	1.8468
56	1.5048	1.7100	1.8468	1.5048	1.7100	1.8468	1.5048	1.7100	1.8468
57	1.5048	1.7100	1.8468	1.5048	1.7100	1.8468	1.5048	1.7100	1.8468
58	2.2572	2.5650	2.7702	2.2572	2.5650	2.7702	2.2572	2.5650	2.7702
59	2.2572	2.5650	2.7702	2.2572	2.5650	2.7702	2.2572	2.5650	2.7702
60	2.2572	2.5650	2.7702	2.2572	2.5650	2.7702	2.2572	2.5650	2.7702

61	3.0096	3.4200	3.6936	3.0096	3.4200	3.6936	3.0096	3.4200	3.6936
62	3.0096	3.4200	3.6936	3.0096	3.4200	3.6936	3.0096	3.4200	3.6936
63	3.0096	3.4200	3.6936	3.0096	3.4200	3.6936	3.0096	3.4200	3.6936
64	0.9900	1.1250	1.2150	0.9900	1.1250	1.2150	0.9900	1.1250	1.2150
65	0.9900	1.1250	1.2150	0.9900	1.1250	1.2150	0.9900	1.1250	1.2150
66	0.9900	1.1250	1.2150	0.9900	1.1250	1.2150	0.9900	1.1250	1.2150
67	1.4850	1.6875	1.8225	1.4850	1.6875	1.8225	1.4850	1.6875	1.8225
68	1.4850	1.6875	1.8225	1.4850	1.6875	1.8225	1.4850	1.6875	1.8225
69	1.4850	1.6875	1.8225	1.4850	1.6875	1.8225	1.4850	1.6875	1.8225
70	1.9800	2.2500	2.4300	1.9800	2.2500	2.4300	1.9800	2.2500	2.4300
71	1.9800	2.2500	2.4300	1.9800	2.2500	2.4300	1.9800	2.2500	2.4300
72	1.9800	2.2500	2.4300	1.9800	2.2500	2.4300	1.9800	2.2500	2.4300
73	0.6098	0.6930	0.7484	0.6098	0.6930	0.7484	0.6098	0.6930	0.7484
74	0.6098	0.6930	0.7484	0.6098	0.6930	0.7484	0.6098	0.6930	0.7484
75	0.6098	0.6930	0.7484	0.6098	0.6930	0.7484	0.6098	0.6930	0.7484
76	0.9148	1.0395	1.1227	0.9148	1.0395	1.1227	0.9148	1.0395	1.1227
77	0.9148	1.0395	1.1227	0.9148	1.0395	1.1227	0.9148	1.0395	1.1227
78	0.9148	1.0395	1.1227	0.9148	1.0395	1.1227	0.9148	1.0395	1.1227
79	1.2197	1.3860	1.4969	1.2197	1.3860	1.4969	1.2197	1.3860	1.4969
80	1.2197	1.3860	1.4969	1.2197	1.3860	1.4969	1.2197	1.3860	1.4969
81	1.2197	1.3860	1.4969	1.2197	1.3860	1.4969	1.2197	1.3860	1.4969

Table 6. Full factorial design L_{81} array for LCC.

Runs	LCC (10^3 USD)								
	y_{31}	y_{32}	y_{33}	y_{34}	y_{35}	y_{36}	y_{37}	y_{38}	y_{39}
1	484.8109	539.5901	576.1095	485.0439	539.8231	576.3425	485.2769	540.0561	576.5755
2	483.5688	538.3475	574.8666	483.7086	538.4873	575.0064	483.8484	538.6271	575.1462
3	483.0364	537.8150	574.3340	483.1363	537.9148	574.4338	483.2361	538.0147	574.5337
4	689.5455	771.7150	826.4947	690.0697	772.2392	827.0190	690.5939	772.7635	827.5432

5	686.7506	768.9192	823.6982	687.0652	769.2337	824.0127	687.3797	769.5482	824.3273
6	685.5528	767.7210	822.4997	685.7775	767.9456	822.7244	686.0022	768.1703	822.9490
7	895.8327	1005.3932	1078.4336	896.7646	1006.3252	1079.3655	897.6966	1007.2571	1080.2975
8	890.8641	1000.4228	1073.4620	891.4232	1000.9820	1074.0211	891.9824	1001.5411	1074.5803
9	888.7346	998.2926	1071.3313	889.1341	998.6920	1071.7307	889.5335	999.0914	1072.1301
10	387.3201	423.0837	446.9261	387.7002	423.4638	447.3063	388.0803	423.8440	447.6864
11	385.2934	421.0563	444.8982	385.5215	421.2844	445.1263	385.7495	421.5125	445.3544
12	384.4248	420.1874	444.0292	384.5877	420.3503	444.1921	384.7506	420.5132	444.3550
13	524.7801	578.4269	614.1915	525.6354	579.2822	615.0468	526.4907	580.1375	615.9021
14	520.2201	573.8653	609.6287	520.7333	574.3785	610.1419	521.2464	574.8916	610.6551
15	518.2658	571.9103	607.6733	518.6323	572.2768	608.0398	518.9989	572.6434	608.4064
16	664.7735	736.3044	783.9917	666.2940	737.8250	785.5122	667.8146	739.3455	787.0328
17	656.6668	728.1948	775.8802	657.5791	729.1071	776.7925	658.4914	730.0195	777.7048
18	653.1925	724.7193	772.4038	653.8441	725.3709	773.0554	654.4958	726.0226	773.7071
19	350.3559	372.4971	387.2579	350.9568	373.0980	387.8587	351.5577	373.6988	388.4596
20	347.1525	369.2925	384.0525	347.5130	369.6530	384.4130	347.8735	370.0135	384.7735
21	345.7796	367.9191	382.6788	346.0371	368.1766	382.9363	346.2946	368.4341	383.1938
22	441.5404	474.7543	496.8969	442.8923	476.1062	498.2488	444.2443	477.4582	499.6008
23	434.3326	467.5439	489.6848	435.1438	468.3551	490.4960	435.9550	469.1663	491.3071
24	431.2436	464.4538	486.5939	431.8230	465.0332	487.1733	432.4024	465.6126	487.7527
25	536.7291	581.0172	610.5426	539.1326	583.4207	612.9460	541.5360	585.8241	615.3495
26	523.9153	568.1988	597.7211	525.3574	569.6409	599.1632	526.7995	571.0829	600.6053
27	518.4237	562.7052	592.2262	519.4538	563.7353	593.2562	520.4838	564.7653	594.2863
28	493.7285	549.0753	585.9731	493.9431	549.2898	586.1877	494.1577	549.5044	586.4022
29	492.5845	547.9308	584.8283	492.7132	548.0595	584.9570	492.8420	548.1883	585.0858
30	492.0941	547.4403	584.3377	492.1861	547.5322	584.4296	492.2781	547.6242	584.5216
31	700.2380	783.2588	838.6060	700.7208	783.7416	839.0889	701.2036	784.2245	839.5717
32	697.6638	780.6837	836.0303	697.9535	780.9734	836.3200	698.2432	781.2631	836.6097

33	696.5605	779.5801	834.9264	696.7675	779.7870	835.1333	696.9744	779.9939	835.3403
34	908.1775	1018.8730	1092.6700	909.0359	1019.7314	1093.5284	909.8943	1020.5898	1094.3867
35	903.6012	1014.2950	1088.0909	904.1162	1014.8100	1088.6059	904.6312	1015.3251	1089.1209
36	901.6399	1012.3330	1086.1284	902.0077	1012.7009	1086.4963	902.3756	1013.0688	1086.8642
37	394.0745	430.1219	454.1535	394.4302	430.4775	454.5091	394.7858	430.8331	454.8647
38	392.1786	428.2253	452.2564	392.3920	428.4387	452.4698	392.6054	428.6520	452.6831
39	391.3661	427.4125	451.4434	391.5185	427.5649	451.5958	391.6709	427.7173	451.7482
40	532.1667	586.2390	622.2872	532.9668	587.0391	623.0873	533.7669	587.8392	623.8874
41	527.9008	581.9716	618.0188	528.3809	582.4517	618.4989	528.8610	582.9318	618.9790
42	526.0726	580.1428	616.1895	526.4156	580.4857	616.5324	526.7585	580.8286	616.8753
43	672.6287	744.7268	792.7922	674.0511	746.1493	794.2147	675.4736	747.5717	795.6371
44	665.0450	737.1404	785.2040	665.8985	737.9939	786.0575	666.7519	738.8473	786.9109
45	661.7949	733.8891	781.9519	662.4045	734.4987	782.5615	663.0141	735.1083	783.1711
46	356.9470	379.3718	394.3217	357.5111	379.9359	394.8857	358.0751	380.5000	395.4498
47	353.9397	376.3634	391.3125	354.2781	376.7018	391.6510	354.6166	377.0403	391.9894
48	352.6508	375.0741	390.0229	352.8926	375.3158	390.2647	353.1343	375.5576	390.5064
49	448.5592	482.1985	504.6246	449.8284	483.4676	505.8938	451.0976	484.7368	507.1629
50	441.7928	475.4296	497.8541	442.5543	476.1911	498.6156	443.3158	476.9526	499.3771
51	438.8928	472.5286	494.9524	439.4368	473.0725	495.4964	439.9807	473.6165	496.0403
52	543.9306	588.7856	618.6889	546.1869	591.0419	620.9452	548.4432	593.2982	623.2015
53	531.9013	576.7520	606.6525	533.2551	578.1058	608.0062	534.6089	579.4596	609.3600
54	526.7459	571.5947	601.4940	527.7129	572.5617	602.4610	528.6799	573.5287	603.4279
55	508.0765	562.0040	597.9557	508.2727	562.2002	598.1519	508.4689	562.3964	598.3481
56	507.0305	560.9576	596.9090	507.1482	561.0753	597.0267	507.2659	561.1930	597.1445
57	506.5822	560.5091	596.4605	506.6663	560.5932	596.5445	506.7504	560.6773	596.6286
58	709.0760	789.9680	843.8960	709.5175	790.4095	844.3374	709.9589	790.8509	844.7789
59	706.7225	787.6136	841.5410	706.9874	787.8785	841.8059	707.2522	788.1433	842.0707
60	705.7138	786.6046	840.5317	705.9030	786.7938	840.7209	706.0922	786.9830	840.9101

61	911.3831	1019.2400	1091.1446	912.1679	1020.0248	1091.9294	912.9527	1020.8096	1092.7142
62	907.1990	1015.0544	1086.9580	907.6699	1015.5253	1087.4289	908.1408	1015.9962	1087.8997
63	905.4058	1013.2606	1085.1637	905.7422	1013.5969	1085.5001	906.0785	1013.9333	1085.8364
64	414.2580	449.7375	473.3905	414.5646	450.0441	473.6970	414.8712	450.3506	474.0036
65	412.6236	448.1025	471.7551	412.8076	448.2864	471.9390	412.9915	448.4704	472.1230
66	411.9232	447.4018	471.0542	412.0545	447.5332	471.1856	412.1859	447.6646	471.3170
67	549.4516	602.6719	638.1521	550.1413	603.3616	638.8418	550.8311	604.0514	639.5316
68	545.7741	598.9931	634.4724	546.1880	599.4070	634.8863	546.6019	599.8208	635.3002
69	544.1981	597.4165	632.8955	544.4937	597.7121	633.1911	544.7893	598.0077	633.4867
70	686.6881	757.6500	804.9579	687.9144	758.8763	806.1842	689.1406	760.1025	807.4104
71	680.1505	751.1100	798.4164	680.8862	751.8458	799.1521	681.6220	752.5815	799.8879
72	677.3486	748.3071	795.6128	677.8742	748.8327	796.1384	678.3997	749.3582	796.6639
73	396.9670	418.8239	433.3951	397.4698	419.3267	433.8979	397.9726	419.8294	434.4007
74	394.2866	416.1425	430.7131	394.5883	416.4442	431.0148	394.8899	416.7458	431.3164
75	393.1378	414.9933	429.5637	393.3533	415.2088	429.7791	393.5688	415.4243	429.9946
76	485.4764	518.2635	540.1215	486.6076	519.3947	541.2528	487.7388	520.5259	542.3840
77	479.4454	512.2303	534.0869	480.1241	512.9091	534.7657	480.8029	513.5878	535.4444
78	476.8607	509.6447	531.5007	477.3455	510.1295	531.9855	477.8303	510.6143	532.4703
79	577.3363	621.0548	650.2005	579.3473	623.0659	652.2115	581.3584	625.0769	654.2226
80	566.6145	610.3292	639.4723	567.8211	611.5358	640.6790	569.0278	612.7425	641.8856
81	562.0195	605.7325	634.8745	562.8814	606.5944	635.7364	563.7432	607.4563	636.5983

To enable an optimization-based solution search, the three output responses obtained from the L_{81} experimental array are transformed into individual desirability functions. Each desirability function maps a response value onto a normalized scale ranging from 0 to 1, where smaller response values correspond to higher desirability. In this study, all three output responses are to be minimized; therefore, equation (15) is employed to construct the desirability function for each response.

To identify optimal solutions, the exponent of each individual desirability function is varied from 0.1 to 2.0 with an increment of 0.1. The **overall desirability** is then defined as the geometric mean of the individual desirabilities and is used as a scalar objective function for optimization. All computations and optimization procedures are implemented in **MATLAB**. By varying the exponents of the individual desirability functions, different overall desirability surfaces are obtained, as illustrated in Figure 2. A higher value of the overall desirability function indicates a more favorable optimal solution.

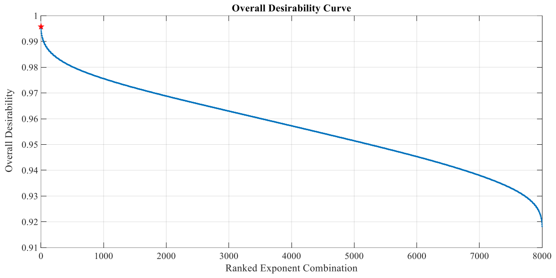

Figure 2. Overall desirability functions.

The optimal solution is obtained when the exponent of each individual desirability function is set to 0.1, resulting in an overall desirability value of 0.9957. Therefore, the optimal design parameters for the 22-kV overhead power transmission line in this case are as follows:

- ✓ Conductor type: ACSR;
- ✓ Size: 240 mm^2;
- ✓ Span: 100 m;
- ✓ Pretension: 70%
- ✓ Maximum sag: 0.3691 ± 0.0223 m;
- ✓ Power loss: 0.6926 ± 0.0612 kW;
- ✓ LCC: 36571.6667 ± 1608.5526 USD.

The main effects plots showing the influence of the four control factors on the three output responses are presented in Figures 3, 4, 5, 6, 7, and 8.

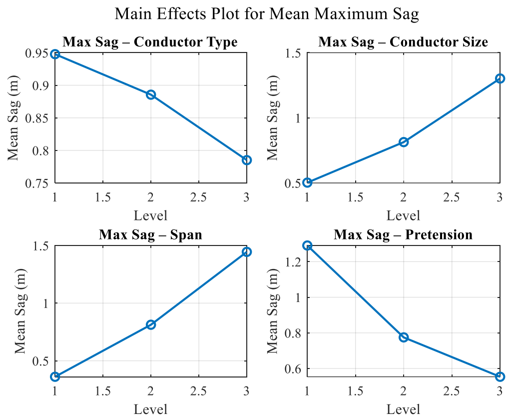

Figure 3. Main effects plot for mean maximum sag.

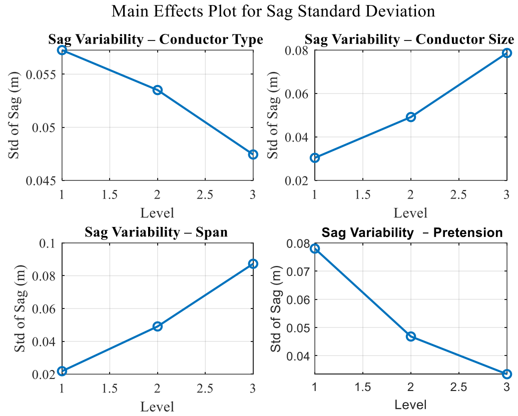

Figure 4. Main effects plot for standard deviation maximum sag.

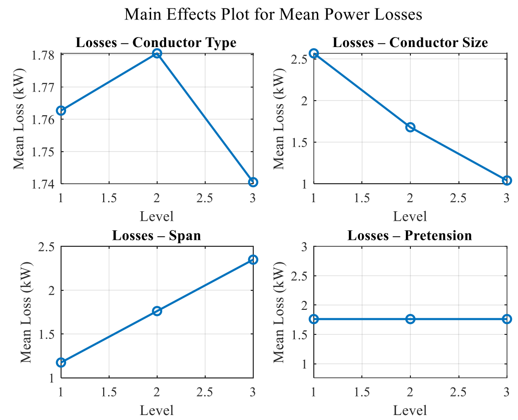

Figure 5. Main effects plot for mean power losses.

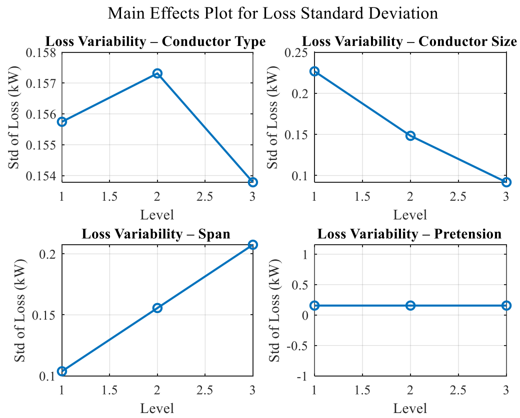

Figure 6. Main effects plot for standard deviation power losses.

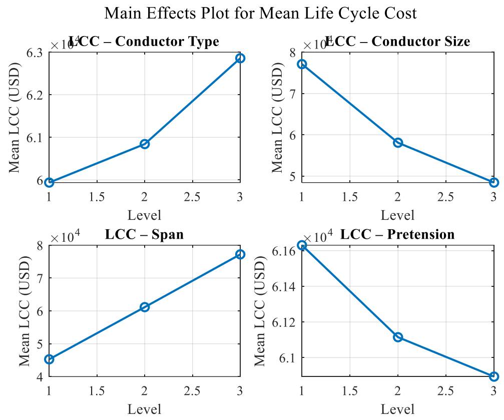

Figure 7. Main effects plot for mean LCC.

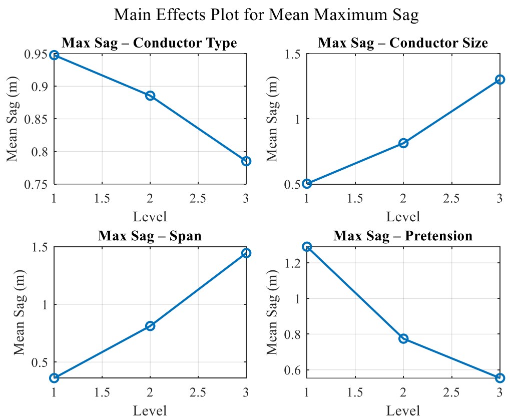

Figure 8. Main effects plot for standard deviation LCC.

Figures 3–8 collectively present the main effects of the control factors on the robust performance of the 22-kV overhead transmission line design. Figures 3, 5, and 7 show the main effects of conductor type, conductor size, span length, and pretension on the mean values of maximum sag, power losses, and LCC, respectively, where each point represents the average response at a given factor level under all noise conditions. In all three responses, span length exhibits the steepest trends, indicating that it is the dominant parameter influencing both mechanical and economic performance, which is physically consistent with the quadratic dependence of sag and resistive losses on span. Pretension also demonstrates a significant influence, particularly in reducing sag and stabilizing LCC, confirming its role in enhancing mechanical robustness. By contrast, conductor type and conductor size show more moderate but non-negligible effects, reflecting their indirect influence through mass, resistance, and material cost.

Figures 4, 6, and 8 show the main effects of conductor type, conductor size, span length, and pretension on the standard deviation values of maximum sag, power losses, and LCC, respectively, where each point represents the standard deviation response at a given factor level under all noise conditions. In all three responses, span length also exhibits the steepest trends, indicating that it is the dominant parameter influencing both mechanical and economic performance, which is physically consistent with the quadratic dependence of sag and resistive losses on span. Pretension also demonstrates a significant influence, particularly in reducing sag and stabilizing LCC, confirming its role in enhancing mechanical robustness. By contrast, conductor type and conductor size show more moderate but non-negligible effects, reflecting their indirect influence through mass, resistance, and material cost.

4. CONCLUSIONS

This paper proposes a robust design–based two-stage optimization framework for overhead power transmission line design by integrating a full factorial L_{81} crossed-array experimental design with desirability function–based multi-objective optimization. The framework explicitly incorporates both control factors and environmental noise factors, enabling robust design solutions that account for uncertainties in ambient temperature and wind speed. A case study on a 22-kV overhead transmission line demonstrated that the proposed approach can effectively minimize maximum sag, power loss, and LCC simultaneously. The results showed that high overall desirability values can be achieved through appropriate selection of desirability function parameters, leading to balanced and robust design solutions. The MATLAB-based implementation confirms the practical applicability of the proposed methodology for transmission line design optimization.

Future work may extend the proposed framework by considering additional design and environmental factors, such as tower configuration, ice loading, and solar radiation. Moreover, efficient experimental designs and advanced metaheuristic optimization techniques can be explored to reduce computational effort and further improve solution quality. The integration of real-time monitoring data is another promising direction for adaptive and data-driven transmission line design and operation.

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