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Xác định sự xuất hiện vết nứt bằng phương pháp biến đổi Wavelet

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TÓM TẮT

Ở bài báo này, độ nhạy của chuyển vị tấm có vết nứt sau khi dùng Phương pháp Phần tử hữu hạn mở rộng (eXtended Finite Element Method - XFEM) và biến đổi Wavelet được phân tích. Sự ảnh hưởng của chiều dài vết nứt và vị trí vết nứt được nghiên cứu. Các giá trị chuyển vị thu được từ bài toán tĩnh được tính toán bằng XFEM và phương pháp Newmark dựa trên mô hình phần tử đẳng tham số tứ giác 4 nút. Các giá trị chuyển vị dùng cho biến đổi Wavelet được nội suy trên cơ sở các hàm dạng, tọa độ và giá trị chuyển vị tại các nút của phần tử tấm. Kết quả khảo sát cho thấy chuyển vị của tấm có vết nứt sau khi biến đổi Wavelet tỏ ra rất nhạy đối với chiều dài và vị trí vết nứt.

Từ khóa: *Extended Finite Element Method (XFEM), tấm có vết nứt, chuyển vị, Wavelet.*

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Identifiation of cracks in structure by Wavelet transform method

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ABSTRACT

In this paper, sensitivity for the transverse displacements of the plate with a crack is analyzed using the Extended Finite Element Method (XFEM) and the Wavelet transform. The effects of the crack length and location are investigated. The transverse displacements obtained from the static analysis are studied by using the XFEM and the Newmark method based on an 4-node quadrilateral isoparametric element. The transverse displacements for transformation are interpolated by the shape functions, the coordinates, and the transverse displacements of the nodes of every element. The examined results show that the transformed transverse displacements are very sensitive to the crack length and location.

Keywords: *Extended Finite Element Method (XFEM), cracked plates, transverse displacements, Wavelet.*

1. INTRODUCTION

Technical diagnostic is an area that attracts many researchers around the world as well as in Viet Nam. Technical diagnostic has a particularly important role to technical objects. Especially, the discovery and assessment of defects and impairments in the structure have been receiving much interest from many authors to research. These impairments have a very large effect on construction capacity and longevity.

There have been many studies of analytical, numerical and experimental methods. Accordingly, the use of numerical methods is very effective in solving the cracked problems. And the Finite Element Method (FEM), has become a tool in use since an early age. However, in some cases FEM has become complicated, such as simulating the movement of non-continuous domains which leads to redivision of the element mesh. The Extended Finite Element Method (XFEM) offers us a new way to model the crack independence of the element mesh; therefore, there is no need to redivide the element grid. Recently, some authors have used XFEM and Wavelet trasforms to study the cracked structures. Bachene et al[1] combined the 9-node isonality element with XFEM to analyze the plate's vibration according to the Mindlin plate theory. Natarajan et al,[2] analyzed natural frequencies of cracked functionally graded material plates by the extended finite element method. Douka et al[3] identified the crack location and depth of the rectangular plates by analyzing Wavelet. Sun. et al[4] analyzed the sensitivity of the transverse displacements of the cracked plate by Wavelet transform.

The paper uses the Wavelet tool to identify cracks in structure. The transverse displacements obtained from the static analysis are studied by using the XFEM and the Newmark method. The transition displacements values found in the paper will be verified using ANSYS software and

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previous published studies to test the accuracy and effectiveness of the proposed method.

2. THEORETICAL FORMULATION

2.1. XFEM Model

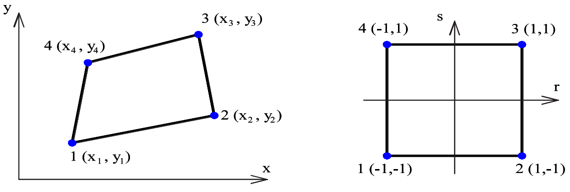
This paper uses the 4-node quadrilateral isoparametric element. The plate element has three degrees of freedom in each node, which are three independent displacements: transverse displacements w and rotating angles β_x, β_y.[5]

The elements of independent displacement quantities are interpolated according to the corresponding node displacements:

$$w=\sum_{i=1}^{4}N_i w_i,\ \beta_x=\sum_{i=1}^{4}N_i\beta_{xi},\ \beta_y=\sum_{i=1}^{4}N_i\beta_{yi} \quad (1)$$

where: w_i, β_{xi}, β_{yi} corresponds to the values of the function w and β_x, β_y at node i, or the degrees of freedom of node i; N_i are the shape functions according to the following natural coordinates:[5]

$$N_1=-\frac{1}{4}(1-r)(1-s),\ N_2=\frac{1}{4}(1+r)(1-s),$$
$$N_3=\frac{1}{4}(1+r)(1+s),\ N_4=\frac{1}{4}(1-r)(1+s) \quad (2)$$



a) Physical co-ordinate system

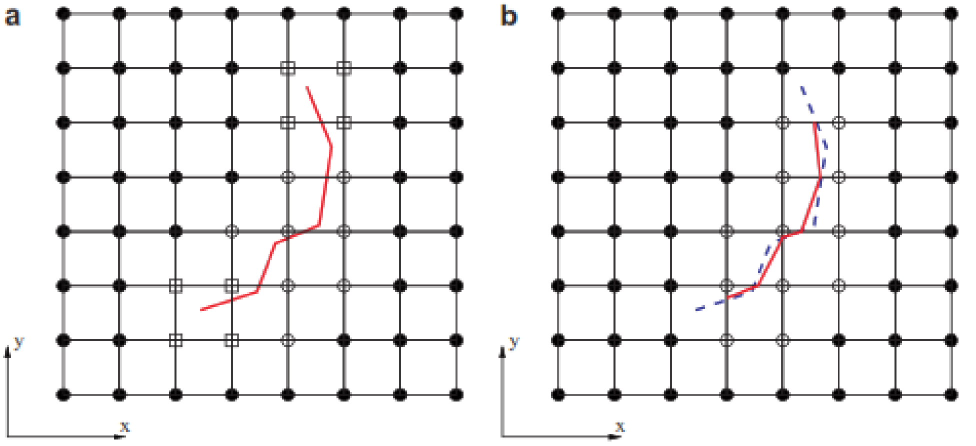
b) Natural co-ordinate system

Figure 1. Isoparametric element in the form of a 4-node quadrilateral

The displacement vector of the node $\mathbf{u}_e$ is made up of 12 components:[6]

$$\mathbf{u}_e^T=\left\{w_1\ \beta_{x1}\ \beta_{y1}...w_4\ \beta_{x4}\ \beta_{y4}\right\}^T \quad (3)$$

The advantage of using enrichment functions in XFEM is that the crack is independent of the element mesh compared to FEM, which means that the element mesh at the crack location must not be redistributed like FEM. For crack modeling, two types of enrichment functions are used: the Heaviside functions and crack-tip functions (Fig. 2).



a) Modeling with crack-tup functions *b) Modeling with Heaviside functions*

Figure 2. Crack modeling using XFEM approach

Examine a point x in the element domain, the displacement approximation at point x is calculated as belows:[6]

$$\mathbf{u}^h(\mathbf{x})=\sum_{i\in\mathbf{N}^{fem}}N_i(\mathbf{x})\mathbf{u}_i+\sum_{j\in\mathbf{N}^{enr}}H(\mathbf{x})N_j(\mathbf{x})\mathbf{a}_j+ \quad (4)$$
$$+\sum_{k\in\mathbf{N}^{asy}}N_k(\mathbf{x})\sum_{\alpha=1}^{4}\mathbf{B}_{\alpha k}(\mathbf{x})\mathbf{b}_{\alpha k}$$

where: $\mathbf{u}^h(\mathbf{x})$ is displacement approximation at point $\mathbf{x}$

$\mathbf{u}_i$ is the vector of degrees of freedom associated with node i

$\mathbf{a}_j$ is the vector of degrees of freedom of the node added by the Heaviside function

$\mathbf{b}_{\alpha k}$ is the vector of degrees of freedom of the node complemented by the asymptote function at the crack-tip

$\mathbf{N}^{fem}$, $\mathbf{N}^{enr}$ $\mathbf{N}^{asy}$ is the set of non-expanding nodes, nodes bisected by cracks and nodes containing crack-tip

$N_i(\mathbf{x}), N_j(\mathbf{x})\, N_k(\mathbf{x})$ and are the corresponding shape functions of the standard element, edge complementer, and crack-tip complement element

$H(\mathbf{x})$ is additional function or Heaviside discontinuous function

$\mathbf{B}_{\alpha k}(\mathbf{x})$ is extension function at crack-tip

Extension function at crack-tip $\mathbf{B}_{\alpha k}$ known as branch functions:

$$\mathbf{B}_{\alpha k} = [B_1, B_2, B_3, B_4] \tag{5}$$

where: $B_1(r,\theta) = \sqrt{r}\sin\left(\frac{\theta}{2}\right)$ $B_2(r,\theta) = \sqrt{r}\cos\left(\frac{\theta}{2}\right)$

$B_3(r,\theta) = \sqrt{r}\sin\theta\sin\left(\frac{\theta}{2}\right)$ $B_4(r,\theta) = \sqrt{r}\sin\theta\cos\left(\frac{\theta}{2}\right)$

where (r, θ) are polar coordinates determined at crack-tip.

2.2. Static equilibrium equation of cracked plate[6]

The system of equations for analysis of static behavior of cracked plate by XFEM:

$$\mathbf{K}.\mathbf{u}^h = \mathbf{f} \tag{6}$$

where: $\mathbf{K}$ is the overall stiffness matrix

$\mathbf{u}^h$ is the vector of nodal freedom

$\mathbf{f}$ is the external load vector. The overall stiffness matrix and the external load vector assembled from element stiffness matrix $\mathbf{K}^e$ and element vector $\mathbf{f}^e$, where:

$$\mathbf{K}_{ij}^e = \begin{bmatrix} \mathbf{K}_{ij}^{\mathbf{uu}} & \mathbf{K}_{ij}^{\mathbf{ua}} & \mathbf{K}_{ij}^{\mathbf{ub}} \\ \mathbf{K}_{ij}^{\mathbf{au}} & \mathbf{K}_{ij}^{\mathbf{aa}} & \mathbf{K}_{ij}^{\mathbf{ab}} \\ \mathbf{K}_{ij}^{\mathbf{bu}} & \mathbf{K}_{ij}^{\mathbf{ba}} & \mathbf{K}_{ij}^{\mathbf{bb}} \end{bmatrix} \tag{7}$$

$$\mathbf{f}_i^e = \left\{ \mathbf{f}_i^{\mathbf{u}} \quad \mathbf{f}_i^{\mathbf{a}} \quad \mathbf{f}_i^{\mathbf{b1}} \quad \mathbf{f}_i^{\mathbf{b2}} \quad \mathbf{f}_i^{\mathbf{b3}} \quad \mathbf{f}_i^{\mathbf{b}4} \right\}^T \tag{8}$$

$$\mathbf{u}^h = \left\{ \mathbf{u} \quad \mathbf{a} \quad \mathbf{b_1} \quad \mathbf{b_2} \quad \mathbf{b_3} \quad \mathbf{b_4} \right\}^T \tag{9}$$

Where

$$\mathbf{K}_{ij}^{rs} = \int_{\Omega^e} (\mathbf{B}_i^r)^T \mathbf{D}_b \mathbf{B}_j^s d\Omega + \int_{\Omega^e} (\mathbf{S}_i^r)^T \mathbf{D}_s \mathbf{S}_j^s d\Omega \tag{10}$$

$$(r, s = \mathbf{u}, \mathbf{a}, \mathbf{b})$$

Include the standard element (**uu**), Heaviside extension element (**aa**), extension element at crack-tip (**bb**), $\mathbf{D}_b$ is the matrix of elastic coefficients due to bending, $\mathbf{D}_s$ is the matrix of elastic coefficients due to shear, **B** is bending deformation matrix, and **S** is shear deformation matrix.

3. CALCULATING CRACKED PLATE DISPLACEMENT USING XFEM

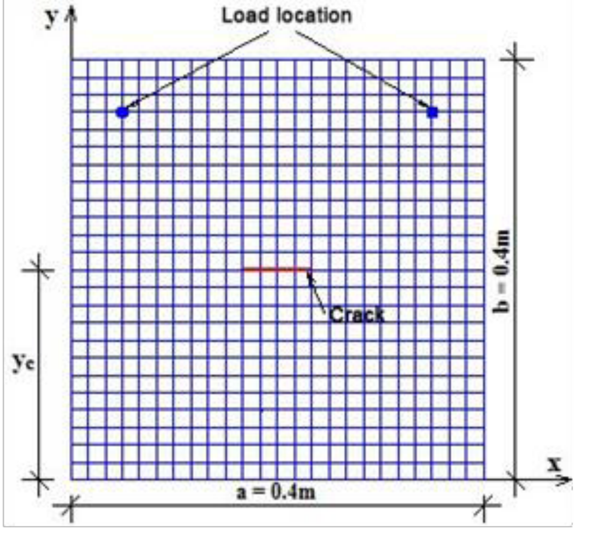


Figure 3. Static plate problem model

Problem 1: Examine a bending thin plate with a crack (Fig. 3), a single with geometric parameters, material properties and load: the plate length *a = 0.4 m*; the plate width *b = 0.4 m*, the plate thickness *h = 0.0005 m*, the crack is in symmetry through the axis *x = 0.2 m*, and parallel to the axis y, the crack location $y_c = 0.2$ *m*, crack length changes *c = 0.04, 0.06, 0.08, 0.10, 0.12 m*; Elastic modules $E = 71\times10^9$ N/m^2, Poisson coefficient $\mu = 0.3$; concentrated load *P = 2.5 N*, located at the coordinates (*x = 0.05 m, y = 0.35 m*) and (*x = 0.35 m, y = 0.35 m*).

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Problem modeling with mesh 25×25. The maximum displacement calculation results for the plate are written in the Matlab equation with the displacement location at node 388. To test the reliability, the results is compared with the result of the solution using the ANSYS program, through the use of the SHELL281 element to simulate a cracked plate.

Table 1. Maximum displacement (w_{max} (×10^{-3} *m*)) of cracked plate

Crack length *c* (*m*)	0	0.04	0.06	0.08	0.10	0.12
w_{max} -FEM (ANSYS) (×10^{-3} *m*)	1.4398	1.4655	1.4750	1.4884	1.5043	1.5300
w_{max} -FEM [7] (×10^{-3} *m*)	-	1.427	-	1.426	-	1.427
w_{max} -XFEM (MATLAB) (×10^{-3} *m*)	1.4388	1.4395	1.4398	1.4534	1.4691	1.4806
Percentage of error with ANSYS	0.0695	1.7741	2.3864	2.3515	2.3400	3.2288
Percentage of error with FEM [7]	-	0.8760	-	1.9215	-	3.7561

From the results shown in Table 1, when the crack length increases by *0.04, 0.06, 0.08, 0.10, 0.12 m* the maximum displacement of the plate also increases. This corresponds to the physical nature of the problem model (like the crack length increases, plate stiffness decreases causing displacement to increases)

Comparison to the results between XFEM(MATLAB) and FEM(ANSYS) for Problem 1 with the crack length changes, it can be seen that the error result depends on the crack length. Specifically, the longer the crack is, the greater the error: the maximum error is 3.2288% when crack length $c = 0.12$ *m.*

Comparison to the the results between XFEM and FEM[7] for Problem 1 with the crack, length changes, it can be seen that the error result also depends on the crack length. The longer the fracture is, the greater the error. The maximum error is 3.7561% when crack length $c = 0.12$ *m*.

The results of these calculations indicate that the program written in MATLAB for a cracked plate is completely reliable.

Problem 2: The same problem parameters as Problem 1. Crack length of the plate $c = 0.04$ *m*; the crack location varies by y_c *= 0.05, 0.10, 0.20, 0.30, 0.35 m.*

The maximum displacement result (node position 338), is shown in Table 2.

Table 2. Maximum displacement (w_{max} (×10-3 m)) of cracked plate

Crack location y_c (m)	0.05	0.10	0.20	0.30	0.35
w_{max} –ANSYS (×10^{-3} *m*)	1.4537	1.4532	1.4655	1.4445	1.4448
w_{max} –MATLAB (×10^{-3} *m*)	1.4388	1.4387	1.4395	1.4422	1.4404
Error (%)	1.0250	0.9978	1.7741	0.1592	0.3045

From the results shown in Table 2, the error between Matlab and Ansys programs was relatively low (1.7741% at the largest). This result shows the high reliability of Matlab program.

4. CRACKED PLATE DISPLACEMENT THROUGH WAVELET TRANSFORM

The following problems are taken from the figures in Section 3 respectively, the set containing 961 displacement values of the plate

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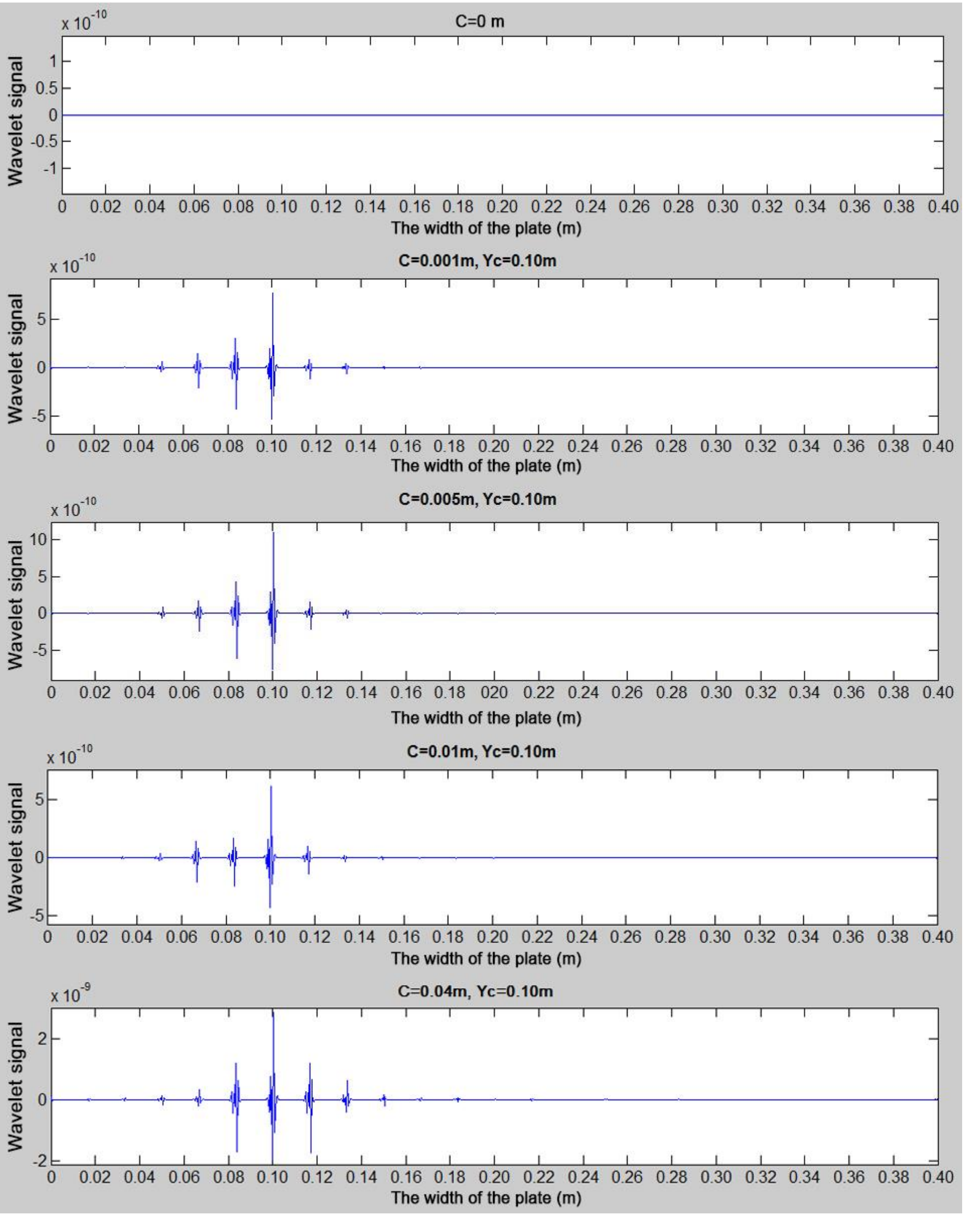
used to transform Wavelet along the axis y, at the location x = 0.2. The author uses Wavelet *sym4* in the symlets kinship to identify cracks in the structure.

Analysis results

During the process of continuous Wavelet transformations, it is possible to effectively observe the sensitivity of the signal at the crack location of the plate.

Problem 1: A four-segments symply support bending plate and the concentrated load $P = 2.5\ N$, located at the coordinates ($x = 0.05\ m$, $y = 0.35\ m$) and ($x = 0.35\ m$, $y = 0.35\ m$); crack location of the unchanged fissure ($y_c = 0.10\ m$), crack length changes (c = 0.001, 0.005, 0.01, 0.04, 0.06, 0.08, 0.10, 0.12 m).



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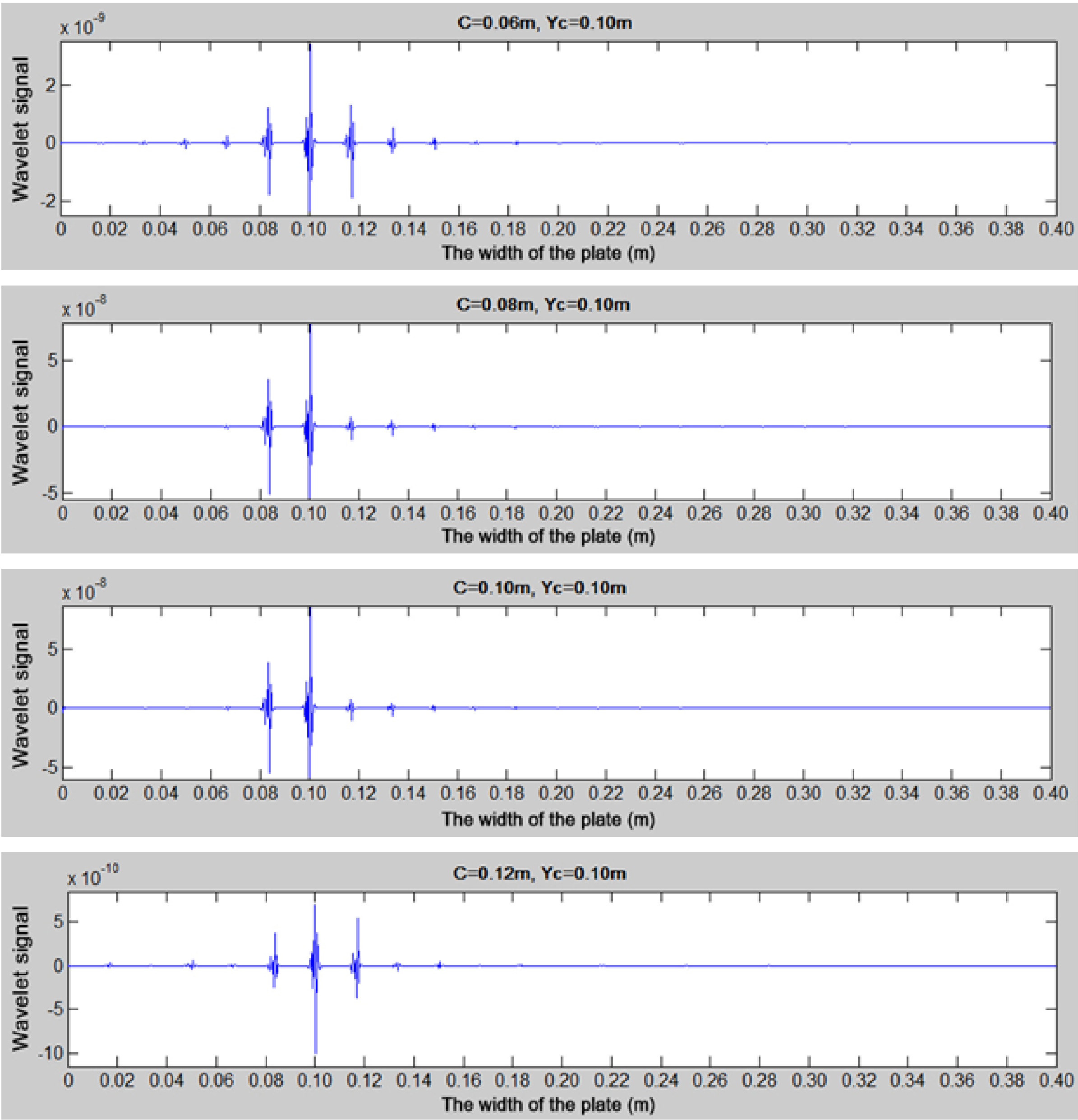
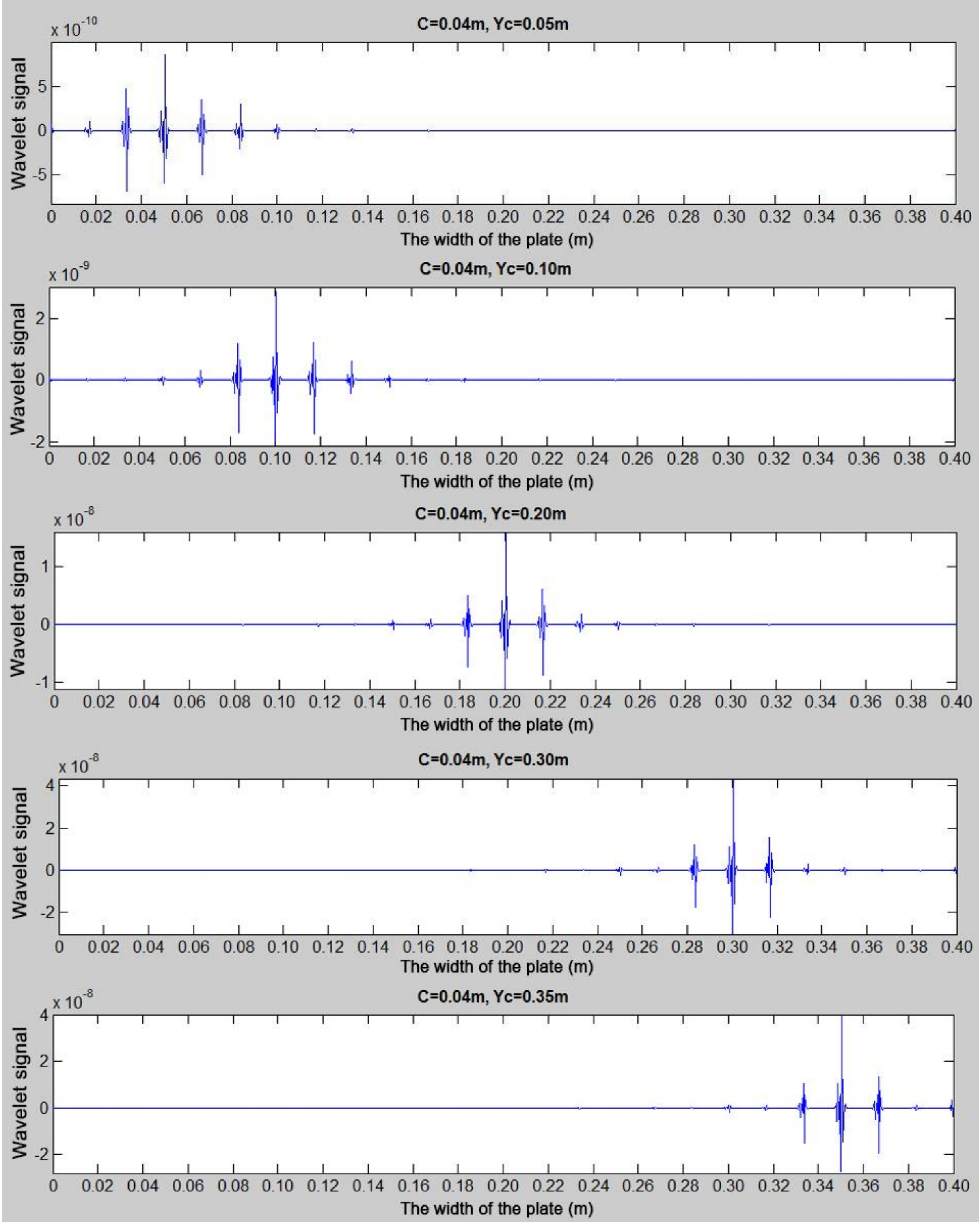


Figure 4. Graph of wavelet signals of static load plate displacement when crack length changes

Problem 2: A four-sided unbending plate and the load $P = 2.5$ N, located at the coordinates ($x = 0.05$ m, $y = 0.35$ m) và ($x = 0.35$ m, $y = 0.35$ m);

Case 1: The crack length is constant ($c = 0.04$ m), the crack location changes ($y_c = 0.05, 0.10, 0.20, 0.30, 0.35$ m).

Case 2: The crack length is constant ($c = 0.005$ m), the crack location changes ($y_c = 0.033, 0.20, 0.367$ m).


C=0.04m, Yc=0.05m
x 10^-10
Wavelet signal
The width of the plate (m)
C=0.04m, Yc=0.10m
x 10^-9
Wavelet signal
The width of the plate (m)
C=0.04m, Yc=0.20m
x 10^-8
Wavelet signal
The width of the plate (m)
C=0.04m, Yc=0.30m
x 10^-8
Wavelet signal
The width of the plate (m)
C=0.04m, Yc=0.35m
x 10^-8
Wavelet signal
The width of the plate (m)

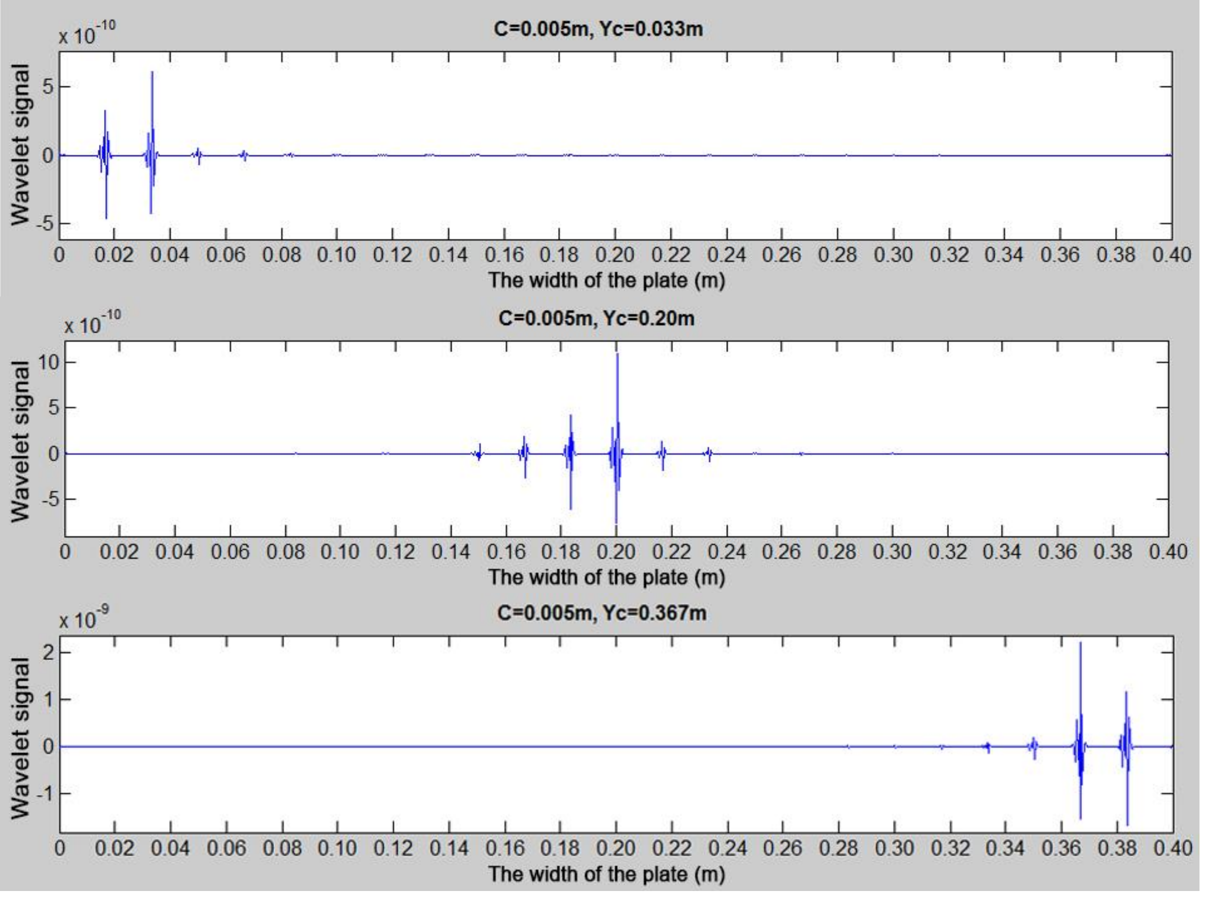


Figure 5. Graph of wavelet signals of static load plate displacement when crack position changes

The sudden signal change is shown clearly in different crack lengths and different crack locations (Fig. 5). In particular, as the crack length increases 0.001, 0.005, 0.01, 0.04, 0.06, 0.08, 0.10, 0.12 *m* the "leap" of the error signal increases: 1.3×10^{-9}, 1.7×10^{-9}, 1.2×10^{-9}, 5×10^{-9}, 6×10^{-9}, 130×10^{-9}, 150×10^{-9}, 170×10^{-9} *m* and the error signal also increases at the crack locations: 0.05, 0.10, 0.20, 0.30, 0.35 *m* (the crack length $c = 0.04$ *m*) and the crack locations: 0.033, 0.20, 0.367 *m* (the crack length $c = 0.005$ *m*). When the crack is located at the load location the signal changes greater and more clearly. When the crack length is small ($c = 0.05$ *m*) and far from the load location ($y_c = 0.033$ *m*), the Wavelet signal graph shows exact crack location. But when the crack length is very small ($c=0.001$ *m*), the error signal at the crack location is still very clear (Fig. 4)

5. CONCLUSIONS

The paper showed XFEM's ability to efficiently apply and Wavelet transforms to identify cracks in the cracked plate. The analysis results show that the displacement computing program by XFEM is very reliable. The examined results show that the transformed transverse displacements are very sensitive to the crack length and location. This opens up the possibility of developing the problem of any cracked plate, developing cracks in the complex dynamic load-bearing plate that will be presented in later studies.

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