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Ứng xử của dầm Timoshenko trên nền đàn nhớt phi tuyến chịu tải trọng di động

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TÓM TẮT

Bài báo nghiên cứu ứng xử động lực học của dầm Timoshenko trên nền đàn nhớt phi tuyến bậc ba chịu tải trọng di động. Dầm hữu hạn có tiết diện không đổi được mô tả theo lý thuyết dầm Timoshenko. Mô hình nền là nền đàn nhớt phi tuyến bậc ba với sáu thông số độc lập. Vì vậy, ứng xử của chuyển vị phi tuyến bậc ba, tác động đồng thời của thông số cắt biến dạng dầm và thông số cắt biến dạng nền đều được kể đến. Phương pháp Galerkin và phép cầu phương tích phân được áp dụng để biến đổi hệ phương trình vi phân chủ đạo thành hệ phương trình vi phân thường. Nghiệm của bài toán là chuyển vị của dầm theo thời gian được xác định bằng phương pháp tích phân từng bước trên toàn miền thời gian Newmark. Kết quả số chỉ ra ảnh hưởng của các thông số đến tốc độ hội tụ của chuỗi Galerkin và chuyển vị của dầm.

Từ khóa: *Dầm Timoshenko, phi tuyến, nền đàn nhớt, tích phân số.*

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Response of Timoshenko beam on nonlinear viscoelastic foundation subjected to a moving load

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ABSTRACT

The paper studies the dynamic response of a Timoshenko beam resting on a third order nonlinear viscoelastic foundation subjected to a moving load. Basing on the theory Timoshenko, a single beam with a constant cross-section is described. The foundation modal is taken as the third order nonlinear viscoelastic foundation with six independent parameters. Therefore, responses of the third order nonlinear deflection, the effects at the same time of the shear deformable beams and the shear deformation of foundation are considered. The Galerkin method and considering integral quadrature method are utilized to transform differential governing equations of motion into the ordinary differential equations. The numerical integration Newmark method is used to solve differential equations and root of equations which is deflection of beam is determined. The numerical results show the dependence of the convergence rate of the Galerkin truncation and the vertical deflection of the beam on system parameters.

Keywords: *Timoshenko beam, nonlinear, viscoelastic foundation, numerical integration.*

1. INTRODUCTION

The analysis about the response of beams resting on different types of foundations subjected to moving load is one of things attracting researches all over the word. This study is used to model, investigate and estimate response of popular structures in civil, industry, traffic… such as bridges, railway, airport pavements, transversally supported pipelines and so on. The available choice model foundation depends on mechanical soil and interaction between beam and foundation in practice. Winkler foundation model has mathematical simplicity but it is mostly considered to represent the elastic foundation[7]. However, Winkler foundation model does not accurately represent the continuous characteristics of foundations because the interaction between the lateral springs is not taken into account in this model. This disadvantage is overcome by including second parameter foundation that indicates the interaction among the linear elastic springs, is seted on stretched membrane; beam or plate with flexural rigidity; shear layer. For instance, Filonenko-Borodich foundation model, Hetenyi foundation model or Pasternak foundation model[6,9] are used.

With the development of the studies on dynamic response of beams resting on a linear foundation, researchers began to pay attention to vibration of elastic beams resting on nonlinear viscoelastic foundation. T.Dahlberg[3] obtained some results arising from moving load and found that the nonlinear model simulated the beam deflection fairly well as compared to measurements, where the linear model did not. That is to say, the influence of foundation's nonlinear cannot be omitted. Ding

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et al[4] investigated the dynamic response of infinite Timoshenko beams lying on nonlinear foundation subjected to a moving concentrated force, using the Adomian decomposition method and perturbation method.

In this paper, the dynamic response of a finite Timoshenko beam on a third order nonlinear viscoelastic foundation are numerically determined via the Galerkin method. The responses of third order nonlinear deflection, the effects at the same time of the shear deformable beams and the shear deformation of foundations are considered, as well as the viscoelasticity.

2. EQUATION OF MOTION

The system under investigation includes beam, foundation, and load, as shown in Fig. 1. Consider a homogeneous beam with a constant cross-section A, a moment of inertial I, a length L, a density ρ, a modulus of elasticity E, a shear modulus G, and an effective shear area $k'A$. F_0, V represent respectively the magnitude of the load and the load speed. The foundation is taken as a nonlinear Pasternak foundation with linear plus cubic stiffness and viscous damping as follows:

$$P_f(X,T) = k_1 U(X,T) + k_3 U^3(X,T) + \mu \frac{\partial U(X,T)}{\partial T} - G_p \frac{\partial^2 U(X,T)}{\partial X^2} \quad (1)$$
$$M_f(X,T) = k_f \psi(X,T) + c_f \frac{\partial \psi(X,T)}{\partial T}$$

Where P_f, M_f represent the force, moment included by the foundation per unit length of the beam, k_1, k_3 are the linear and nonlinear foundation parameters, respectively. G_p, μ are the shear deformation coefficient and the damping coefficient of the foundation, respectively, T is the time, X, U are the spatial coordinate along the axis of the beam and the vertical displacement function, respectively.

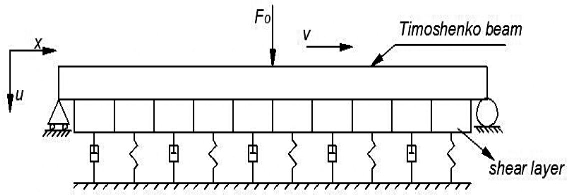

Figure 1. The model of a finite Timoshenko beam on a nonlinear viscoelastic Pasternak foundation

Using the Timoshenko beam theory and considering load balancing, the governing differential equation of motion for the beam developed as:

$$\rho A \frac{\partial^2 U}{\partial T^2} + k'AG\left[\frac{\partial \psi}{\partial X} - \frac{\partial^2 U}{\partial X^2}\right] + k_1 U +$$
$$k_3 U^3 + \mu \frac{\partial U}{\partial T} - G_p \frac{\partial^2 U}{\partial X^2} = F_0 \delta(X - VT) \quad (2)$$
$$\rho I \frac{\partial^2 \psi}{\partial T^2} - EI \frac{\partial^2 \psi}{\partial X^2} + k'AG\left[\psi - \frac{\partial U}{\partial X}\right] + k_f \psi + c_f \frac{\partial \psi}{\partial T} = 0$$

Where k_f, c_f are foundation rocking stiffness and damping coefficients, $\psi(X,T)$ is the slope function due to bending of the beam, $\delta(X - VT)$ is the Dirac delta function used to deal with the moving load concentrated load.

The Galerkin truncation method[1] is used to discretize the system and the series expansion forms for $u(x,t)$ and $\psi(x,t)$ with simply supported condition are assumed as:

$$u(x,t) = \sum_{k=1}^{\infty} q_k(t)\phi_k(x), \quad \phi_k(x) = \sin(k\pi x)$$
$$\psi(x,t) = \sum_{k=1}^{\infty} \xi_k(t)\upsilon_k(x), \quad \upsilon_k(x) = \cos(k\pi x) \quad (3)$$

Where $\phi_k(x)$, $\upsilon_k(x)$ are the trial functions, $q_k(t)$, $\xi_k(t)$ are sets of generalized displacements.

Weight function is taken as trial function itself for the Galerkin method. With the normal orthonormal condition, the simply supported boundary conditions lead to the following equations:

$$\int_0^1 \phi_k(x) w_i(x) dx = \begin{cases} 0 & k \neq i \\ 1/2 & k = i \end{cases}$$
$$\int_0^1 \psi_k(x) w_i(x) dx = \begin{cases} 0 & k \neq i \\ 1/2 & k = i \end{cases} \quad (4)$$

Using Eq (4) and considering Differential Quadrature method[2] yields

$$\ddot{q}_i(t) + \mu \dot{q}_i(t) + \left[k_1 + G_p (k\pi)^2 + \alpha (k\pi)^2\right] q_i(t)$$
$$+2k_3 \sum_{j=1}^{N} I_j F_j w_i(x_j) - \alpha(i\pi)\xi_i(t) = 2F_0 w_i(vt) \quad (5)$$
$$\ddot{\xi}_i(t) + c_f \dot{\xi}_i(t) + \left[k_f + (k\pi)^2 + \beta\right] \xi_i(t) - \beta(i\pi) q_i(t) = 0$$
$$i = 1, 2, \ldots, n$$

The above mentioned ordinary differential equations can be solved via the numerical integration Newmark method and roof of equations which is deflection of beam are determined.

3. NUMERICAL RESULTS

Numerical examples are given for investigating model truncation convergence and the effect of parameters are displayed in this part. The physical and geometrical properties of the Timoshenko beam, foundation and the moving load are listed in Table 1.

Table 1. Properties of the beam, foundation and load

	Item	Value	Dimensionless value
Beam	Young's modulus E / GPa	6.998	-
	Shear modulus G / GPa	77	
	Mass density $\rho / kg.m^{-3}$	2373	-
	Height of pavement h / m	0.3	-
	Width of pavement b / m	1.0	-
	Length L / m	160	-
	Shear coefficients k'	0.4	-
	α	-	4.401
	β	-	1.502×10^7
Foundation	Linear stiffness k_1 / MPa	8	97.552
	Nonlinear stiffness k_3 x $MN.m^{-4}$	8	2.497×10^6
	Viscous damping $\mu / MN.s.m^{-2}$	0.3	39.263
	Shear deformation coefficient G_P / N	6.669×10^7	0.0318
	Rocking stiffness k_f / N	10^8	1.626×10^5
	Rocking damping coefficient $c_f / N.s$	1.5×10^6	2.618×10^4
Movinng load	Load F_0 / N	2.126×10^6	1.01×10^{-4}
	Speed $V / m.s^{-1}$	20	0.01165

The vertical deflection of beam center with time is shown in Fig. 2. The Galerkin truncation term is set to five cases, namely $n = 25$, $n = 40$, $n = 45$, $n = 50$, $n = 60$. There are large differences between the $n = 25$, with the $n = 45$, and there are discernible differences between the $n = 40$, with the $n = 45$. Moreover, the results of $n = 45$, $n = 50$, $n = 60$ are almost the same. Therefore, the $n = 45$, is accurate enough for analyzing the dynamic response of beam. The results are in agreement with reference[8] and the results showed that the convergence rate of equations root is faster when comparing the numerical integration Newmark method and the fourth-order Runge-Kutta method ($n = 45 < n = 150$).

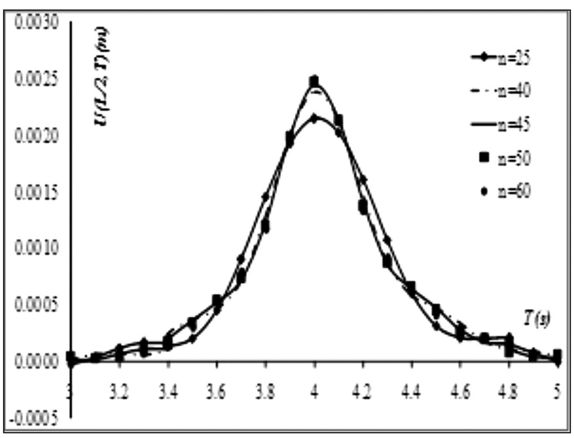

Figure 2. The vertical deflection of the beam's midpoint

The effects of the shear modulus of a beam and the shear deformation coefficient of its viscoelastic nonlinear foundation on the deflections of the beam are illustrated in Fig.3 and Fig.4, respectively. It is clear that the maximum value of the dynamic deflection occurs almost at the mid-span of the beam. As seen from the simulation results, the biggest deflections decrease with the increase of shear modulus of beams and the increase of shear deformation coefficient of foundations. It should be noted that a Timoshenko beam model gradually become a Euler-Bernoulli beam model when value of G decreases. So the maximum deflection of a Timoshenko beam is much smaller than a Euler-Bernoulli beam. This conclusion corresponds with the results of Ref[4]. Moreover, with value $G_P = 0$, a Pasternak foundation turns into a Winkler foundation. That is to say, the maximum deflection of a Timoshenko beam on a Pasternak foundation is much smaller than that of a beam on a Winkler foundation. It is noted that this conclusion corresponds with the result of Ref.[5]

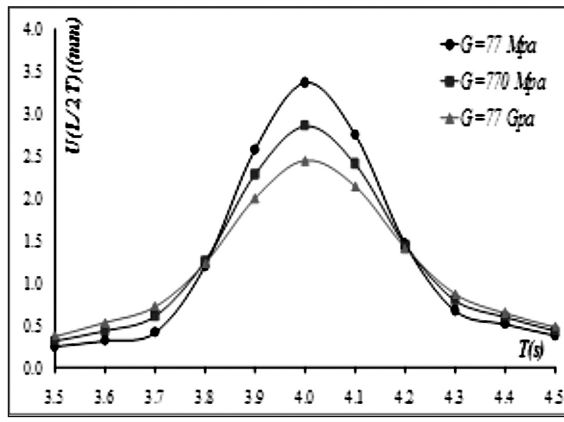

Figure 3. Effects of the shear modulus of the beam on the deflection of the beam

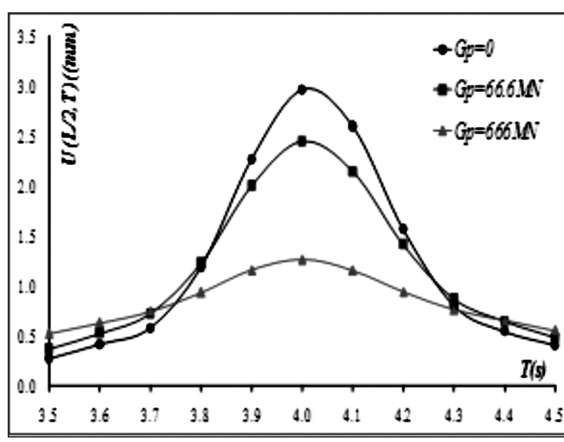

Figure 4. Effects of the shear deformation coefficient of the foundation on the deflection of the beam

4. CONCLUSIONS

This paper is built a model problem includes: a finite Timoshenko beam; A nonlinear viscoelastic with many independent parameters; And moving load. The governing different equations of motion for the beam is developed using the load balancing and the Timoshenko beam theory. Basing on the Galerkin method and considering the numerical integration method are used to solve differential equations. The root of equations which is deflection of beam are determined. The numerical investigation shows the effect of the shear modulus of beam and the shear deformation coefficient to the dynamic response of the beam and the convergence of the Galerkin truncation.

REFERENCES

1. C.A.J. Fletcher. *Computational Galerkin Method,* Springer-Verlag, Newyork, 1984.
2. C. Shu. *Differential Quadrature and Its Application in Engineering,* Springer-Verlag, London, 2000.
3. T. Dahlberg. Dynamic interaction between train and nonlinear railway track model, *Proceedings of the Fifth European Conference on Structural Dynamics,* **2002**, *2, 1155-1160*.
4. H. Ding, K.L. Shi, L.Q. Chen, S.P. Yang. Dynamics response of an infinite Timoshenko beam on a nonlinear viscoelastic foundations to a moving load, *Nonlinearm Dynamics*, **2013**, *73, 285-298*.
5. H. Ding, L.Q. Chen, S.P. Yang. Convergence of Galerkin truncation for dynamic response of finite beams on nonlinear foundations under a moving load, *Journal of Sound and Vibration,* **2012,** *331, 2426-2442*.
6. M.H. Kargarnovin, D. Younesian. Dynamics of Timoshenko beams on Pasternak foundations under moving loads, *Mechanical Research Communications,* **2004**, *31, 713-723.*
7. D. Thambiratnam, Y. Zhuge. Dynamic analysis of beams on an elastic foundation subjected to moving loads, *Journal of Sound and Vibration*, **1996**, *198, 149-169*.
8. Y. Yang, H. Ding, L.Q. Chen. Dynamics response to a moving load of a Timoshenko beam resting on a nonlinear viscoelastic foundations, *Acta Mechanica Sinica*, **2013**, *29(5), 718 - 727*.
9. D. Younesian, M.H. Kargarnovin. Response of the beams on random Pastermak foundations subjected to harmonic moving loads, *Journal of Mechanical Science and Technology,* **2009**, *23, 3013-3023*.