

Report on

**Splitting piecewise cubic Bézier curves and the equivalence constants  
for some norms**

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The article investigates the splitting of uniform  $N$ -piece cubic Bézier curves into  $2N$ -piece curves while maintaining the same degree. The primary contribution is the derivation of equivalence constants for norms defined by control points ( $\|\cdot\|_p^{B_{N,3}}$  and  $\|\cdot\|_p^{B_{2N,3}}$ ) on the space  $B_{N,3}$ . These constants are shown to be independent of the number of pieces  $N$ , which is a useful result for trajectory optimization and checking the convergence of curve sequences.

All the proofs seem to be correct. The presentation is clear. A carefully revised version of this paper can be recommended for publication in Quy Nhon University Journal of Science with only minor checks required.

**Comments:**

- Page 2, line 1: “equivalence relations”  $\rightarrow$  “equivalence constants”.
- Page 2, paragraph 1, line 2: “or curves which given”  $\rightarrow$  “or are defined”.
- Page 2, paragraph 2, line 17: “it need”  $\rightarrow$  “it requires”.
- Page 2, paragraph 2, line 20: “can be use”  $\rightarrow$  “can use”.
- Page 3, paragraph 1, line -9: remove “For the readers can follow along easily.”.
- Page 3, paragraph 2, line -9: “For convenient”  $\rightarrow$  “For convenience”.
- Page 4, paragraph 1: What is  $B_m$  in Definition 5?
- Page 4, paragraph 2, line 7: In the proof of Lemma 7, can you explain the first inequality in Case  $p \in [1, \infty[$ ?
  - Page 6, paragraph 1: In Case 1 and 2: “ $\sum_{i=3}^3$ ”  $\rightarrow$  “ $\sum_{i=0}^3$ ”.
  - Page 6, paragraph 2, line 1: “j3+4”  $\rightarrow$  “j3+3”.