Referee's report for the manuscript QNUJS-B2608
Title "*Explicit Low-Temperature Bounds for the Free Energy of the Blume-Capel Model*"

The paper is devoted to investigate phase transitions and microscopic structure of matter in the spirit of Blume-Capel model using Pigorov-Sinai theory. Such a model is used to describe many phenomina in condensed matter physics. The major contribution of the paper is to provide some sharper lower bounds for temperature arguments, which significantly enlarges the scope of the phase analysis.

The manuscript is highly technical and relies on a large number of complex mathematical expressions. While the complexity of the derivations makes a line-by-line verification exhaustive, the internal consistency of the proofs and the clarity of the presentation are commendable. The manuscript is well-written and, in my opinion, it is suitable for publication after some minor revisions are addressed. Here I list in the sequel several typos and issues.

- *Page 2, 2nd sentence of Introduction*: here set of spins is $\{-1, 0, +1\}$ while in other places $\{-1, 0, 1\}$.
- *Page 4, Definition 3.1*: $\# \in \{+, -, 0\}$
- *Page 5, Definition 3.5*: $x \in \partial_{ex} A_j$ is opposite to equation (13) $\partial^{ex} A = \cdots$
- *Page 5, Definition 3.7*: typo $\bar{\gamma}' \subset \text{int}\gamma$ (another notation int $(\gamma) = \cdots$before)
- *Page 7, paragraph before Definition 3.10*: ... may be found in [8?]
- *Page 9, equation (46)* : typo $\sum_{\gamma \in \Gamma, x \in \gamma} w(\gamma), e^{a|[\bar{\gamma}]_1|} \leq \cdots$

Summary. The manuscript could be accepted under minor revision. The author should do by themselves to cover the typos before publication.